HANDOUT 8.1b: Sample unit for Grade 7

Geometry and measures 1

UNIT 7.2
9 hours

About this unit

This unit is the first of four units on geometry and measures for Grade 7.

This unit is designed to guide your planning and teaching of mathematics lessons. It provides a link between the standards for mathematics and your lesson plans, and should help you to plan the content, pace and level of difficulty of lessons. You will need to adapt it to meet the needs of your class.

Expectations

By the end of the unit, most students will identify alternate, supplementary and corresponding angles and know angle properties related to diagonals of squares, rectangles, parallelograms and rhombuses. They will use these and other properties to find the values of unknown angles in geometric figures. They will use a ruler and compasses to construct angle bisectors and perpendicular bisectors and, together with a protractor, to construct simple geometric figures from given data.

Students who progress further will calculate interior and exterior angles of polygons. They will solve problems using angle and symmetry properties of polygons and angle properties of parallel and intersecting lines. They will construct 2-D shapes from given information, including scale drawings.

Students who make less progress will recognise vertically opposite angles, angles on a straight line and around a point. They will know the sum of the angles of a triangle and the relationship between the exterior angle of a triangle and its interior opposite angles. They will use these and other properties to find the values of angles in geometric figures. They will use a ruler and protractor to construct triangles, given two sides and the included angle, or two angles and the included side.

Resources

The main resources needed for this unit are:

- overhead projector (OHP)
- sharp pencil, straight edge (ruler), compasses and protractor for each student
- dynamic geometry system (DGS), computer and data projector
- *Sample lesson plans for mathematics*: lesson plan 7.3

Key vocabulary and technical terms

Students should understand, use and spell correctly:

- names of common two-dimensional shapes: *circle, isosceles triangle, equilateral triangle, scalene triangle, quadrilateral, square, rectangle, parallelogram, trapezium, kite, rhombus, regular/irregular polygon, pentagon, hexagon, octagon*
- types of angles: *acute, obtuse, reflex, base angles, interior/exterior angles, supplementary, complementary, alternate, corresponding*
- properties of lines: *parallel, diagonal, perpendicular, bisector, hypotenuse*
- words related to problem solving and reasoning: *classify, solve, explain, justify, demonstrate, prove, proof*

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Objectives for the unit

Unit 7.2

9 hours	SUPPORTING STANDARDS including Grade 6 standards	CORE STANDARDS Grade 7 standards	EXTENSION STANDARDS including Grade 8 standards
3 hours **Properties of shapes**	6.8.3 Use the labelling conventions for angles, lines and geometric figures. 6.8.4 Identify angles in a straight line, at a point and vertically opposite angles.	7.9.1 Identify, sketch, label and describe angle, side, diagonal and symmetry properties of plane shapes: • triangles (isosceles, equilateral, right-angled, acute- and obtuse-angled scalene triangle); • quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, kite); • polygons (pentagon, hexagon, octagon, decagon).	8.6.2 Identify reflection and rotation symmetry properties of 2-D shapes, including triangles, quadrilaterals and regular polygons, and draw 2-D symmetrical figures.
3 hours **Angles, shapes and geometric reasoning**	6.8.6 Identify equal lengths or find unknown angles in geometric figures, involving: • angles in a straight line, around a point or vertically opposite angles; • the angle sum of a triangle; • side or angle properties of an isosceles, equilateral, right-angled and scalene triangle; • side or angle properties of a square, rectangle or parallelogram.	7.9.2 Calculate unknown angles in geometric figures, involving: • angles in a straight line, around a point or vertically opposite angles; • corresponding, alternate and supplementary angles; • side or angle properties of isosceles, equilateral, right-angled and scalene triangles, including the angle sum and exterior angle properties; • side or angle properties of squares, rectangles, parallelograms and rhombuses, including angle properties related to their diagonals; • angle bisectors and perpendicular bisectors. 7.1.3 Present and explain solutions and conclusions in the context of the original problem, orally and in writing. 7.1.4 Use logical reasoning to establish the truth of a statement.	8.6.1 Calculate interior and exterior angles of polygons. 8.6.3 Use knowledge of angle properties of intersecting and parallel lines, and of the angle, side and symmetry properties of triangles, quadrilaterals and polygons, to conjecture or deduce properties in a given figure. 8.1.4 Present a concise, reasoned argument orally and in writing. 8.1.5 Use step-by-step reasoning to deduce properties or relationships in a given geometrical figure.
3 hours **Constructions**	6.8.8 Use ruler, protractor and set square to: • measure and draw angles; • draw perpendicular and parallel lines; • draw rectangles and squares. 6.8.7 Use a ruler and protractor to construct a triangle: • given two angles and the included side; • given two sides and the included angle.	7.9.4 Use a ruler, set square, protractor and compasses to: • measure and draw line segments and angles; • draw parallel and perpendicular lines; • draw circles and arcs; • construct angle bisectors and perpendicular bisectors; • construct simple geometric figures from given data. 7.9.5 Use ICT to generate and explore constructions.	8.6.8 Use ruler, set square, protractor and compasses to construct geometrical figures from given data, on paper and using ICT.

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Activities

Unit 7.2

Objectives	Possible teaching activities	Notes	School resources
Properties of shapes **3 hours** Identify, sketch, label and describe angle, side, diagonal and symmetry properties of plane shapes: • triangles (isosceles, equilateral, right-angled, acute- and obtuse-angled scalene triangle); • quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, kite); • polygons (pentagon, hexagon, octagon, decagon). Present and explain solutions and conclusions in the context of the original problem, orally and in writing.	**Short activities** Tell the class to imagine two equilateral triangles of the same size, and to place them together, edge to edge. Ask for the name of the shape that is formed (a rhombus). Ask students to explain why the answer is always the same (the same shape is formed no matter which edges are placed together). Repeat with two right-angled triangles (to form a square, an isosceles triangle, a kite, two different parallelograms). This time different shapes are formed when equal edges are placed together.	After the visualisation activity, demonstrate how to combine the shapes using paper shapes placed on an OHP. This activity can be repeated using three or four equilateral triangles.	This column is blank for schools to note their own resources, e.g. textbooks, worksheets.
	Tell the class to imagine a quadrilateral that has only two lines of symmetry. Ask a student to draw one possible shape on the board with its name. Ask another student to draw another example. (The two possible shapes are a rectangle and a rhombus.) Tell the class that the shape must now have equal sides, and ask them which of the two quadrilaterals satisfies both this condition and the symmetry condition (the rhombus does).	This activity can be repeated using different quadrilaterals.	
	Give each student a set of six quadrilaterals cut out from thin card: a square, a rectangle, a parallelogram, a rhombus, a kite, a trapezium. Ask students to hold up the shapes that have: • no lines of symmetry (parallelogram, trapezium) • one line of symmetry (kite) • two lines of symmetry (rectangle, rhombus) • four lines of symmetry (square) • no rotation symmetry (kite, trapezium) • rotation symmetry of order two (rectangle, parallelogram, rhombus) • rotation symmetry of order four (square)	This activity can be extended to include identifying shapes with one pair of parallel sides, two pairs of parallel sides, and so on.	
	Quadrilaterals Draw each of the following shapes on the board or OHP. Ask the class to describe all the mathematical properties of each shape. List all the properties below each and tell the students to copy these into their books. • Square: four equal sides, four right angles, opposite sides parallel, diagonals bisect each other at right angles, four lines of symmetry, rotation symmetry of order four • Rectangle: two pairs of equal sides, four right angles, opposite sides parallel, diagonals bisect each other, two lines of symmetry, rotation symmetry of order two • Parallelogram: two pairs of equal sides, two pairs of equal angles, opposite sides parallel, diagonals bisect each other, no lines of symmetry, rotation symmetry of order two • Rhombus: four equal sides, two pairs of equal angles, opposite sides parallel, diagonals bisect each other at right angles, two lines of symmetry, rotation symmetry of order two • Kite: two pairs of adjacent sides of equal length, one pair of equal angles, diagonals intersect at right angles, one line of symmetry, no rotation symmetry • Trapezium: one pair of parallel sides, no rotation symmetry (an isosceles trapezium has one line of symmetry)	The quadrilaterals activity can be repeated for triangles and polygons.	Lesson plan 7.3

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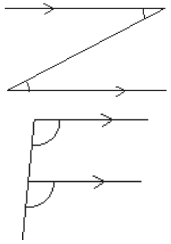
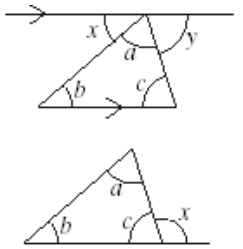
Objectives	Possible teaching activities	Notes	School resources
Angles, shapes and geometric reasoning **3 hours** Calculate unknown angles in geometric figures, involving: • angles in a straight line, around a point or vertically opposite angles; • corresponding, alternate and supplementary angles; • side or angle properties of isosceles, equilateral, right-angled and scalene triangles, including the angle sum and exterior angle properties; • side or angle properties of squares, rectangles, parallelograms and rhombuses, including angle properties related to their diagonals; • angle bisectors and perpendicular bisectors. Present and explain solutions and conclusions in the context of the original problem, orally and in writing. Use logical reasoning to establish the truth of a statement.	**Short activity** Use a target board such as the one on the right. The theme is numbers which sum to 180 or 90. Ask students to: • choose a number from the board, and subtract it from 180; • find two numbers that add up to 180; • add two numbers together to give a total less than 180, and subtract the result from 180; • find two numbers that add up to 90; • find three numbers that add up to 180; • find a set of numbers with a sum of 360; Ask students when it is useful to be able to use the skills that they have been practising (e.g. when calculating angles in a triangle, on a straight line, or around a point). Give one or two examples of finding an unknown angle when two out of three angles in a triangle, on a straight line or around a point are known.	<table><tr><td>90</td><td>70</td><td>50</td><td>30</td><td>60</td><td>58</td><td>73</td></tr><tr><td>45</td><td>105</td><td>32</td><td>17</td><td>127</td><td>15</td><td>165</td></tr><tr><td>63</td><td>87</td><td>25</td><td>120</td><td>148</td><td>20</td><td>3</td></tr><tr><td>163</td><td>135</td><td>75</td><td>110</td><td>130</td><td>65</td><td>40</td></tr></table> The target board activity can be repeated using different sets of numbers or different rules.	
	Alternate and corresponding angles Draw a letter Z (with rotation symmetry of order two) on a piece of card. Ask what happens to the Z when the card is rotated through 180°. Ask students to be specific. Demonstrate, by using an acetate sheet with a tracing of the letter on it, that the top and bottom angles are equal. Ask what happens to the two angles if a letter Z is drawn without the top and bottom lines being parallel. Use geo-strips to demonstrate. Ensure that students recognise that if the lines are parallel, then the angles are equal, but if they are not parallel, then the angles will be different. Point out that the correct name for pairs of angles like this is *alternate* angles. Use geo-strips or a letter F drawn on card to establish the rules for corresponding angles.		
	Angle sum and exterior angle of triangle Remind students that the sum of the angles of a triangle is 180°. Ask them how they can show this. A possible answer will be: 'Draw a triangle and measure the three angles'. Explain that this is not accurate; it only demonstrates that the rule works for particular examples. Demonstrate that the rule works by cutting off the corners of a triangle and placing the three angles together to form a straight line. Explain that this demonstration only shows the rule; there could be triangles for which the rule does not work. This is why a proof is needed to convince us that the rule works for every triangle.		
	Show a proof that the sum of the angles of a triangle is 180°. To prove that $a + b + c = 180°$, draw a line parallel to one side of the triangle. Let x and y be the other two angles formed on the line with a. Then $x = b$ (alternate angles), $y = c$ (alternate angles), and $a + x + y = 180°$ (angles on a line). So $a + b + c = 180°$. Show a proof that the exterior angle of a triangle is equal to the sum of the two interior opposite angles: x is an exterior angle of the triangle. To prove $a + b = x$, let the other interior angle of the triangle be c. Then $a + b + c = 180°$ (angles in a triangle) and $x + c = 180°$ (angles on a straight line). So $a + b = x$.		Lesson plan 7.3

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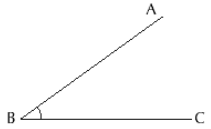
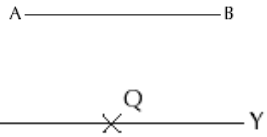
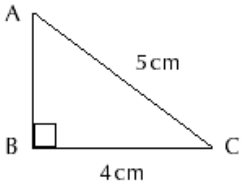
Objectives	Possible teaching activities	Notes	School resources
Constructions **3 hours** Use a ruler, set square, protractor and compasses to: • measure and draw line segments and angles; • draw parallel and perpendicular lines; • draw circles and arcs; • construct angle bisectors and perpendicular bisectors; • construct simple geometric figures from given data. Present and explain solutions and conclusions in the context of the original problem, orally and in writing.	**Constructions** Show the class how to complete the five constructions in the examples below. Explain that these constructions are very accurate when done with a sharp pencil, which is why they are used in other subjects such as technology. **Example 1: To construct the mid-point and the perpendicular bisector of the line AB** • Draw a line segment AB of any length. • Set compasses to any radius greater than half the length of AB. Draw two arcs with the centre at A, one above and one below AB. • With compasses set at the same radius, draw two arcs with the centre at B, to intersect the first two arcs at C and D. • Join C and D to intersect AB at X. X is the mid-point of the line AB. The line CD is the perpendicular bisector of the line AB. **Example 2: To construct the bisector of the angle ABC** • Draw ∠ABC of any size. • Set compasses to any radius. With centre B, draw an arc to intersect BC at X and AB at Y. • With compasses set to any radius, draw two arcs with centres at X and Y, to intersect at Z. • Join BZ. BZ is the bisector of ∠ABC, and ∠ABZ = ∠CBZ. **Example 3: To construct the perpendicular from a point P to a line segment AB** • Set compasses to any suitable radius. Draw arcs from P to intersect AB at X and Y. • With compasses set at the same radius, draw arcs with the centres at X and Y to intersect at Z below AB. • Join PZ. PZ is perpendicular to AB. **Example 4: To construct the perpendicular from a point Q on a line segment XY** • Set compasses to a radius that is less than half the length of XY. With the centre at Q, draw two arcs on either side of Q to intersect XY at A and B. (The line XY may need to be extended.) • Set compasses to a radius that is greater than half the length of XY. With the centres at A and B, draw arcs above and below XY to intersect at C and D. • Join CD. CD is the perpendicular from the point Q. **Example 5: To construct the right-angled triangle ABC** • Draw a line BC 4 cm long. • Use the method in Example 4 to construct the perpendicular from B. (Extend the line BC.) • Set compasses to a radius of 5 cm. With centre at C, draw an arc to intersect the perpendicular from B. • The intersection of the arc and the perpendicular is A.	Students will need a sharp pencil, a straight edge (ruler), compasses and a protractor.     
Use ICT to generate and explore constructions.	Use a dynamic geometry system (DGS) to explore constructions.		

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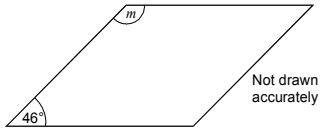
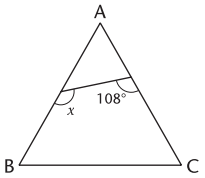
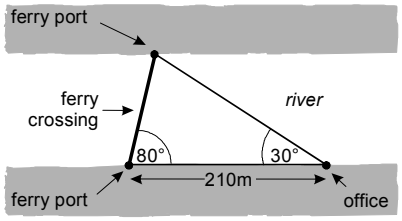
Assessment	Possible assessment activities	Notes	School resources
Students should be able to respond successfully to a range of questions. For example:	*A quadrilateral must be a parallelogram if it has:* A. *one pair of adjacent sides equal* B. *one pair of parallel sides* C. *a diagonal as an axis of symmetry* D. *two adjacent angles equal* E. *two pairs of parallel sides* **TIMMS Grade 8** *The diagram shows a rhombus.* *One angle is 46°.* *Calculate the size of the angle marked m.*  *Triangle ABC is equilateral.* *Calculate the size of angle x.*  *Explain the difference between a demonstration and a proof.* *Use compasses to construct a triangle that has sides 8 cm, 6 cm and 7 cm.* *Here is a plan of a ferry crossing.* *Make an accurate scale drawing of the ferry crossing.* *What is the length of the ferry crossing on your diagram?*  Not drawn accurately		

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