HANDOUT 8.1c: Sample unit for Grade 10 foundation

UNIT 10F.3
6 hours

Algebra 1

About this unit

This unit is the first of four units on algebra for Grade 10 foundation.

This unit is designed to guide your planning and teaching of mathematics lessons. It provides a link between the standards for mathematics and your lesson plans, and should help you to plan the content, pace and level of difficulty of lessons. You will need to adapt it to meet the needs of your class.

Expectations

By the end of the unit, most students will be aware of the role of symbols in algebra. They generate and manipulate formulae. They use functions and begin to appreciate numerical and algebraic applications in the real world. They use mathematics to model and predict the outcomes of real-world applications and conjecture alternative possibilities with 'What if…?' and 'What if not…?' questions.

Students who progress further will recognise when quadratic functions are increasing, decreasing or stationary, and find the tangent at a point on the graph of a function.

Students who make less progress will work on the same areas of mathematics but to a lesser degree of difficulty. They develop an understanding of the core principles of the topic.

Resources

The main resources needed for this unit are:

- graphics calculators
- graph paper

Key vocabulary and technical terms

Students should understand, use and spell correctly:

- *function, domain, range, mapping*
- *equation, identity, formula, variable, example, expression*
- *continuous, discontinuous*

Objectives for the unit

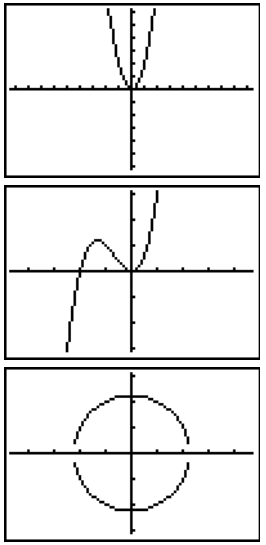
Unit 10F.3

6 hours		SUPPORTING STANDARDS including Grade 9 standards		CORE STANDARDS Grade 10 foundation standards		EXTENSION STANDARDS including Grade 11 foundation standards
2 hours **Formulae, equations and identities**	8.4.2	Simplify and compare algebraic expressions to determine their equivalence.	10.4.5	Distinguish the different roles played by letter symbols in algebra, and understand that the transformation of algebraic objects generalises the well-defined rules of arithmetic. Recognise that letters are used to represent: • the solution set of initially unknown numbers in *equations*; • defined variables in *formulae;* • generalised independent numbers in *identities*; • new equations, expressions or functions in terms of known, or given, expressions or functions.		
	8.4.3	Evaluate linear expressions or formulae by substituting given integer values of the variables.				
	8.4.6	Determine whether given integer values satisfy a given linear equation.				
	10.4.1	Solve any linear equation with one unknown.				
4 hours **Functions, mappings and graphs**	8.5.5	Recognise that functions can be represented as ordered pairs, tables, graphs, words or equations; given a function in one representation, generate an equivalent representation.	10.5.1	Use function notation; investigate a range of mathematical and physical situations to develop the concepts of *function*, *domain* and *range*, recognising one-to-one and many-to-one mappings as functions and a one-to-many mapping as not a function.	11.5.3	Draw the tangent line at a point on the graph of a function, calculate the slope of this line and interpret the behaviour of the function at that point, knowing whether the function is increasing or decreasing at the point, or stationary.
	8.5.1	Use a graphics calculator to generate sequences and plot graphs.	10.5.2	Understand and use the concept of related variables and, in special cases, set up appropriate functional relationships between them.		
			10.5.4	Use a graphics calculator to plot a range of simple functional relationships, some continuous and others discontinuous, arising in familiar contexts.		
			10.5.5	Recognise when a graph represents a functional relationship between two variables and when it does not.		
	10.5.3	Plot a graph to show the relationship between two variables given quantitative information between the variables in tabular or algebraic form.	10.1.2	Use mathematics to model and predict the outcomes of real-world applications; compare and contrast two or more given models of a particular situation.		
			10.1.11	Conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions.		

Activities

Unit 10F.3

Objectives	Possible teaching activities	Notes	School resources
2 hours Distinguish the different roles played by letter symbols in algebra, and understand that the transformation of algebraic objects generalises the well-defined rules of arithmetic. Recognise that letters are used to represent: • the solution set of initially unknown numbers in *equations*; • defined variables in *formulae;* • generalised independent numbers in *identities*; • new equations, expressions or functions in terms of known, or given, expressions or functions.	**Activities** Ask students to work in pairs to form 'dictionary definitions' for *formula*, *equation* and *function*. These should then be shared with the class in order to discuss and develop them further. Students should then discuss questions such as the following: • Distinguish between an expression and an equation. Give some examples. • Why is $5x + 3 = 15$ an equation? What does the x stand for? • What do d, s, and t stand for in $s = d / t$? • Why is $5(x + 2) \equiv 5x + 10$ an identity? • What are the differences between an equation, expression, function and an identity?		
	Encourage students to consider the ways in which algebra is generalised arithmetic. For example, what rules in arithmetic are the same in algebra? Consider the commutative and associative laws. Discuss ways students approach problems, for example: 1 Pick any two consecutive numbers. 2 Square each, and find the difference (ignoring the sign). 3 Add the two original numbers. 4 Explain why the answers in steps 2 and 3 are the same.	Students who are comfortable with variables and symbolic manipulation usually solve the problem using an appropriate algebraic equation, for example, $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$ $= 2n + 1$ $= (n + 1) + n$	
	Set students some conjectures to prove or disprove and ask them to make a note of their approaches to them. • The square of every even number is a multiple of 4. • The square of every odd number is odd. • A square number with a units digit of 1 is the square of a number with a units digit of 1.	Some students may approach the problem initially by considering some numerical examples. This is an acceptable beginning for such problems and is based on an understanding of algebra as generalised arithmetic. It must, however, be superceded by an algebraic approach if the statement is to be proved.	
	Ask students to research the contribution of Arabic scholars to the development of mathematics and to the development of algebra in particular by mathematicians such as Al-Khwarizmi.	An Internet searchwill lead to sites such as: http://www-gap.dcs.st-and.ac.uk/~history/HistTopics/Arabic_mathematics.html http://it.stlawu.edu/~dmelvill/mesomath/	

Objectives	Possible teaching activities	Notes	School resources
4 hours Use function notation; investigate a range of mathematical and physical situations to develop the concepts of *function*, *domain* and *range*, recognising one-to-one and many-to-one mappings as functions and a one-to-many mapping as not a function. Understand and use the concept of related variables and, in special cases, set up appropriate functional relationships between them. Use a graphics calculator to plot a range of simple functional relationships, some continuous and others discontinuous, arising in familiar contexts. Recognise when a graph represents a functional relationship between two variables and when it does not.	**Functions** Provide each pair of students with a set of cards with different inputs and outputs on them. Students should pair these up to form functions. Some suitable combinations are shown opposite. For each function students should find five input/output pairs. Next students should be able to justify to the rest of the class why the pair form a function and whether it is one-to-one or many-to-one. Students should then use the cards to form relationships that are not functions and be able to explain why this is so. For each function, there can only be one output for a given input, while a non-function may have more than one output for the same input. For example, people of more than one age can wear shoes of a certain size.	<table><tr><td>person</td><td>nationality</td></tr><tr><td>identity number</td><td>birth date</td></tr><tr><td>capital city</td><td>county</td></tr><tr><td>side length</td><td>area of square</td></tr><tr><td>a word</td><td>first letter</td></tr></table>	
	Plot some functions on a graphics calculator and explore their properties. • Which represent one-to-one functions? • Which represent many-to-one functions? • For each function state the domain and range Students should also try producing screens on graphics calculators, or sketching possibilities, which are not functions.		
	Explore the graphs of functions arising from familiar contexts, for example: • quantity and price (the more that is bought, the more it costs); • a cooling curve; • distance and time for a constant speed; • depth of water in a tank and time. Include step-wise or discontinuous functions such as: • weight and postage cost; • cost for a job and time taken (for example, a plumber charges the same amount for a job that takes between one hour and two hours); • time zone and longitude.		

Objectives	Possible teaching activities	Notes	School resources
Use mathematics to model and predict the outcomes of real-world applications; compare and contrast two or more given models of a particular situation. Conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions.	Ask students to consider the use of functional relationships in science and the use of related variables in scientific contexts. Students will be familiar with the equations of motion: $s = u + at$ $v^2 - u^2 = 2as$ $d = ut + \frac{1}{2}at^2$ Challenge students to express d without using u, or to eliminate a from $s = u + at$ and $v^2 - u^2 = 2as$. Students will also be familiar with potential energy equations: $U = mgh$ and $U = \frac{1}{2}kv^2$ U can be eliminated from this pair of equations. When in science is this equivalence used? What does it mean in context? If $W = Fd$ and $F = ma$, then express W in terms of d, m and a.	This is a good opportunity for joint work with the science department. Check with the students' science teachers which formulae they are familiar with and use these as a basis for this part of the unit.	

Assessment	Possible assessment activities	Notes	School resources
Students should be able to respond successfully to a range of questions. For example:	*If p is a person, state with reasons whether each of the following maps are functions:* a. *p maps to the place of birth of p;* b. *p maps to brother of p;* c. *p maps to nationality of p;* d. *p maps to teacher of p;* e. *p maps to mother of p.*		
	A firm rents out cars by the day or by the week. The daily charge rate is QR 170 with 150 km free and then QR 2 for every additional kilometre. The weekly charge is QR 1400 with no additional charges. A man needs to hire a car for five days. How many kilometres will he have to drive to make it worthwhile to hire the car for a week?		
	Look up any country in an atlas and pick six towns from it. Which of these maps represents a function and which does not: towns → country; country → towns? Justify your answer. What are the domain and range for the mapping that represents a function?		
	In an electric circuit, $V = IR$, where V is the voltage in volts, I is the current in amps and R is the resistance in ohms. The electrical power in watts is $P = VI$. Find a formula connecting the variables P, V and R.		
	Draw a graph showing the functional relationship between postage rate in Qatar and the weight of package to be posted.		
	Is $(x + 4)^2 = x(x + 12) – 4(x – 4)$ an equation or an identity? Explain your reasoning.		
	Students could prepare a poster or booklet to explain the work from this topic, considering some of the following. • *What are the key points?* • *What do you need to remember?* • *Add some examples that will help someone else to understand the topic.* • *What have you learned?* • *What do you still need help with?*		