Day 1 Session 4 — 90 minutes

The reasoning and problem solving strand

Objectives

By the end of this session teachers will:

- understand the nature of the reasoning and problem solving strand of the mathematics standards and how it relates to the other three strands;
- have begun to consider the implications of the reasoning and problem solving strand for teaching and learning.

Resources

For the trainer

- Computer with data projector, Microsoft PowerPoint and Presentation 4.ppt
- Whiteboard or flipchart
- Spare copies of the evaluation form for Day 1
- *Curriculum Standards for mathematics: Grades K to 12*

For each teacher

- *Teacher's pack*
  Handouts 4.1, 4.2, 4.4 and 4.5
  evaluation form for Day 1
- A copy of Handout 4.3 (to be given out at the relevant point of the session)
- A basic calculator
- *Curriculum Standards for mathematics: Grades K to 12*

Session outline

Introduction Slides 4.1–4.2	Whole group presentation	10 minutes
Some practical activities Slide 4.3 Handouts 4.1–4.4	Practical tasks for small groups Task 1: Sixes are banned Task 2: Some practical activities	50 minutes
Types of problem Slides 4.4–4.10	Whole group presentation and discussion	15 minutes
Action planning Handout 4.5 Evaluation form for Day 1	Task 3: Action points Completion of evaluation form	15 minutes

Introduction

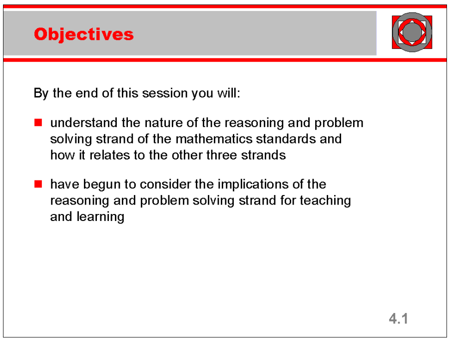
10 minutes

Before the session starts, brief any interpreter about the key points of the session.

Check that each teacher has a copy of the *Sample lesson plans*.

Load **Presentation 4.ppt**.

Explain that this session focuses on the reasoning and problem solving strand of the standards. Use **slide 4.1** to introduce the objectives for this session.

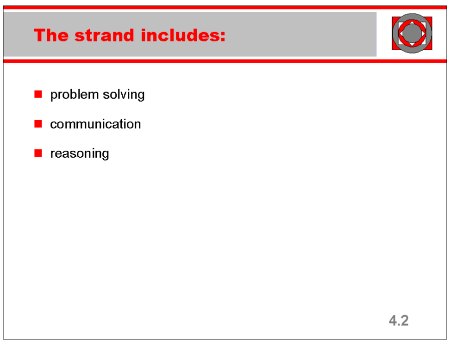


Explain these points.

- Number and algebra, geometry and measures and data handling cover the understanding, knowledge and procedural skills and techniques associated with the content of the mathematics curriculum.
- The reasoning and problem solving strand is different from the other three strands because it is 'content free'. It involves the higher order, generic skills needed to solve mathematical problems.
- The reasoning and problem solving standards apply to each of the content strands. They are taught by integrating them with aspects of the other three strands and are assessed alongside the content strands.
- Because it is integrated it is difficult to identify teaching time for the reasoning and problem solving strand. The proportion of each content strand devoted to reasoning and problem solving should increase steadily from grade to grade. By Grades 10 to 12, problem solving and reasoning should be fostered in all lessons as an essential part of learning mathematics and developing understanding of its applications.

Ask everyone to check the first couple of pages of the standards for one of the grades that they teach, where the approximate proportion of time to be devoted to reasoning and problem solving is stated.

Refer everyone to the first part of the scope and sequence charts for the relevant grades in their school. Ask them to note how the reasoning and problem solving strand is cumulative: the skills acquired in earlier grades continue to be applied and extended in higher grades. Show **slide 4.2**, to summarise the three aspects of the reasoning and problem solving strand.



Say that:

- **problem solving** encompasses a wide range of situations, some purely mathematical and some based on real-world situations;
- **communicating** is about students interpreting, formulating, discussing and recording mathematics and presenting findings;
- **reasoning** is not only about reasoning but includes making and testing hypotheses, spotting patterns, generalising, explaining, justifying and proving.

Stress that the three aspects are intertwined. It would be difficult to solve a problem without reasoning and without any element of communication, even if there were no recording of the solution or presentation of findings.

Say that it will be important to consider to what extent opportunities for problem solving, communication and reasoning are built into the school's scheme of work and incorporated into the teaching of mathematics.

Some practical activities

50 minutes

Say that in this session participants will look at some practical tasks and consider some of the reasoning and problem solving skills involved.

Task 1: Sixes are banned

Tell participants that they will need their calculators. Ask them to work in pairs and to use **Handout 4.1**, Sixes are banned. Explain that they are to imagine that the 6 key on their calculator is broken; 6 can be made to appear in the display but not by pressing the 6 key. Each pair must work out the answers to the calculations by using their calculators. They must agree on the necessary calculator key sequence *before* they carry out the calculation.

Allow up to 6 to 7 minutes for the pairs to complete the activity. Early finishers can be asked to reflect on and discuss their strategies and thinking.

Background notes for trainers

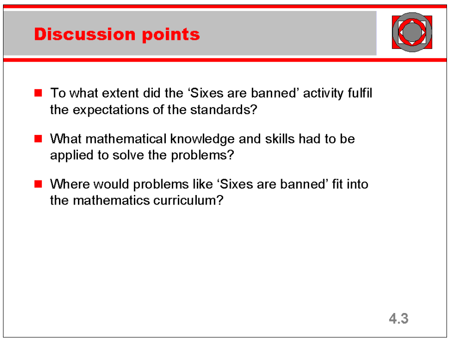
There are many different ways of finding the answers but, for example:

- 32 + 16 can be tackled as 32 + 17 – 1;
- 126 – 58 can be tackled as 127 – 1 – 58;
- 48 × 6 can be tackled as 48 × 3 × 2, or as 24 × 12;
- 146 ÷ 7 can be tackled as (153 – 7) ÷ 7 = (153 ÷ 7) – 1;
- 62 × 16 can be tackled as 31 × 32, or as 124 × 8;
- 263 – 76 can be tackled as 273 – 10 – 75 – 1;
- 263 ÷ 62 can be tackled as 131.5 ÷ 31;
- 36 × 0.6 can be tackled as 4 × 9 × 3 ÷ 5.

Now refer the pairs to **Handout 4.2**, a table showing the three aspects of the strand: problem solving, communicating and reasoning. Ask pairs to write in appropriate columns verbs describing what they did in the 'Sixes are banned' activity. For example, under ‘Communicating’, they may have been *discussing*; under ‘Reasoning’, they may have been *predicting*.

After about 5 minutes, give out **Handout 4.3**. Explain that these are words that other teachers have used (and are not necessarily the right answers). Ask pairs to compare them with their own version, and to add further verbs to Handout 4.3 if they can. Allow a further couple of minutes.

Bring the whole group together. Point out that the ‘Sixes are banned’ activity drew upon each of problem solving, communicating and reasoning, although not necessarily equally. Discuss the points on **slide 4.3** with the whole group.



Make sure that teachers appreciate that, suitably adapted to the types of numbers and calculations students are working on, a problem like this could be integrated into lessons for Grades 5 to 8 involving calculation methods.

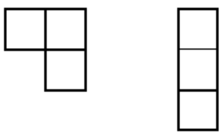
Task 2: Some practical activities

Refer everyone to **Handout 4.4**, Some practical activities. Select activities suited to the teachers in the group and ask them to try them.

Activity 1

Substitute similar activities appropriate for the age of students in the teachers’ schools, if relevant.

Triominoes are shapes made from three identical squares touching edge to edge. There are just two triominoes, discounting reflections and rotations.

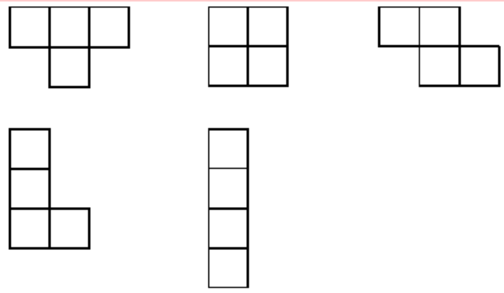


Tetrominoes are shapes made from four identical squares touching edge to edge. How many tetrominoes are there?

Allow a couple of minutes for participants to sketch the possible tetrominoes, then ask someone to describe how they approached the question and how they found their solutions. Ask them how they could prove they had all possible solutions. Ask if anyone else had a different way of approaching this question and a different way of proving they had all possible solutions.

Background notes for trainers

One way of showing that all possible solutions have been found is to place the fourth square in every possible position on the two triominoes and reject duplicates. Only five distinct possibilities can be found. This approach illustrates proof by exhaustion.



Activity 2

Discuss the truth of the statement 'Multiplication makes a number bigger'.

Allow a couple of minutes for participants to work on this in pairs. Take feedback from at least two pairs, asking them to explain how they approached the task. Ask if, as well as proving or disproving the statement, anyone is able to say for which numbers the statement is true.

Background notes for trainers

The fact that $4 \times \frac{1}{2} = 2$ proves that the statement is false. This illustrates the important idea that just one counter-example can disprove a statement. Participants may offer a generalisation:

$a \times b > a$ when $a > 0$ and $b > 1$, or when $a < 0$ and $b < 1$

Activity 3

The perimeter of a triangle is 48 cm. The length of the shortest side is x cm and of another side is $2x$ cm. Prove that $8 < x < 12$.

Give pairs 5 minutes to tackle this activity. Circulate while they are working and identify pairs that are tackling the question in different ways.

Invite one or two pairs to explain the way they approached the question.

Background notes for trainers

Some may use a visual approach while others may use a more formal algebraic proof, using the property that the longest side of a triangle is greater than the sum of the two shorter sides.

Activity 4: Perimeter dots

On square dotty paper, draw some polygons that have only one dot inside them. Investigate the relationship between the number of dots on the perimeter of each polygon and its area.

Now investigate the relationship for polygons with two dots inside them.

If you have time, find a relationship between the area of a polygon with 12 dots on its perimeter and the number of dots inside it.

Ask the pairs to take 6 to 7 minutes to investigate the problem. Circulate while they are working, helping the less confident to get started by drawing simple shapes. When participants think that they have found a relationship for polygons with one internal dot, ask questions such as: 'Are you certain that the relationship you have found will apply to all polygons with one dot inside?' Suggest that they draw some unusual shapes to test their conjecture.

Background notes for trainers

For polygons with one internal dot, $p = 2A$, or $A = {}^1/_2\, p$, where p is the number of dots on the perimeter and A is the area (in square units). For polygons with two internal dots, $p = 2(A – 1)$ or $A = {}^1/_2\,(p + 2)$.

For polygons with 12 dots on the perimeter, $A = d + 5$, where A is the area in square units and d is the number of internal dots.

Now ask everyone to look again at **Handout 4.3** and spend a few minutes in pairs highlighting some of the skills they used while working on the practical activities.

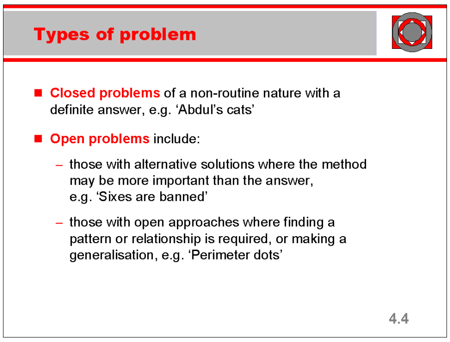
Types of problem

15 minutes

Abdul's cats

Abdul has 3 cats, each a different weight. The first and the second weigh 7 kg altogether, the second and the third weigh 8 kg altogether. The first and the third weigh 11 kg altogether. What is the weight of each cat?

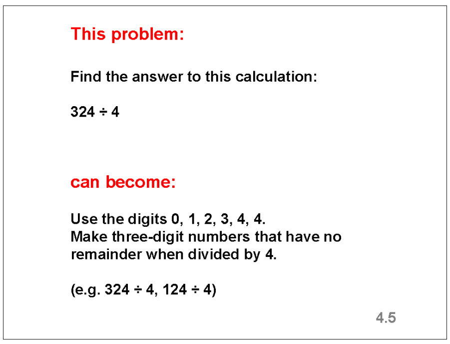
Use **slide 4.4** to discuss the difference between closed and open problems.

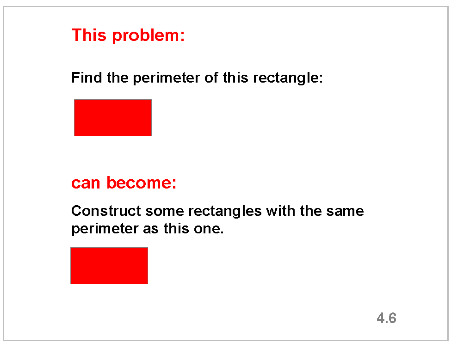


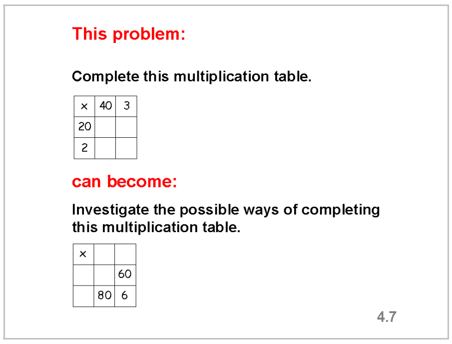
Make these points.

- Investigative approaches (i.e. the verbs on Handout 4.3) can be used for closed and open problems.
- A closed problem can often be transformed into an open problem by extending or modifying it in some way.

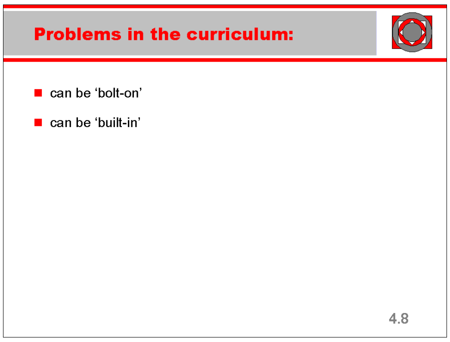
Use **slides 4.5, 4.6 and 4.7** to illustrate the last point.







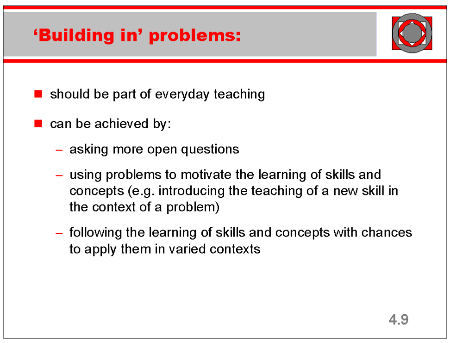
Show **slide 4.8**. Say that it is worth considering for a moment when and how students should experience problem solving and investigational work.

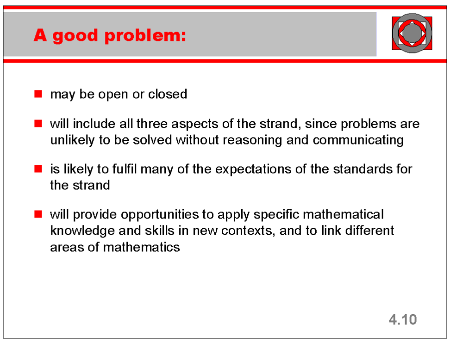


Say that one approach to reasoning and problem solving has been through a series of activities done after the 'content' has been learned. Traditionally, this is provided through problems at the ends of sections or chapters of a published scheme or textbook. Investigations are sometimes tackled as ends in themselves, taught separately from the rest of the mathematics curriculum, sometimes in special lessons.

The alternative to these **'bolt-on'** approaches is making reasoning and problem solving integral to the learning of mathematics, so that it is **'built-in'** throughout each unit of work and is part of everyday teaching. Teachers should 'build-in' wherever possible.

Show **slides 4.9 and 4.10**.





Say that the activities themselves don't determine whether they are built-in or bolt-on. Their relationship to the other strands determines what the options are. Both the 'Sixes are banned' and the other activities could be presented in either way.

Action planning

15 minutes

Task 3: Action points

Refer teachers to **Handout 4.5** in their *Teacher's pack*. Ask them to work together in school groups, to reflect on the day's sessions and to make notes on the actions that they will take to follow them up. Allow about 10 minutes.

Check whether anyone has any unanswered questions from those that they listed at the beginning of the day and answer these now.

Remind everyone to bring their copies of the *Teacher's pack* to the second day of the course.

Thank everyone for their contributions. Invite them to complete the evaluation form for the first day and to give it to you before leaving.