Day 4 Session 12 — 80 minutes

Geometry 2

Objectives

By the end of Sessions 11 and 12 teachers will:

- have explored activities involving angles and shapes;
- have considered how to help students to understand angle;
- have considered how practical work underpins more advanced analytical work in later grades.

Resources

For the trainer

- Computer with data projector, Microsoft PowerPoint and Presentation 12.ppt
- Overhead projector and acetate sheets, and a black arrow made from card
- Interactive teaching program: *Angle*
  (download free from www.standards.dfes.gov.uk/primary/mathematics)
- Smile program: *Angles* (see www.smilemathematics.co.uk)
- Large paper triangle and large paper quadrilateral
- *Curriculum Standards for mathematics: Grades K to 12*

For each teacher

- *Teacher's pack*
  Handouts 12.1–12.3
- *Curriculum Standards for mathematics: Grades K to 12*

For each group

- Masking tape

Session outline

Introduction to angle Slides 12.1–12.2 Handouts 12.1–12.2	Whole group presentation and discussion Task 1: What students should be taught about angle Task 2: Giving instructions	30 minutes
Exploring angles Handout 12.3	Whole group presentation and discussion, paired work Task 3: Using a protractor Task 4: Estimating angles Task 5: Angles in a triangle Task 6: Extension activity	45 minutes
Summary Slide 12.3	Summary	5 minutes

Introduction to angle

30 minutes

Before everyone arrives, brief any interpreter about the key points of the session.

Load **Presentation 12.ppt**.

Say that this session continues the work on geometry started in the previous session. Explain that in this session the focus will be on angle.

Task 1: What students should be taught about angle

Refer everyone to **Handout 12.1**, which states what students should be taught about angle.

Ask teachers to look at the scope and sequence charts for Grades 3, 4, 5 and 6 to identify the work that relates to angle (pages 25 and 31 of the standards). Allow a few minutes for teachers to study the scope and sequence charts.

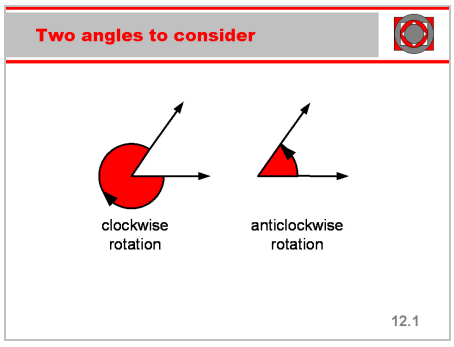
Say that the standards include the concept of angle being a measurement of turn, and ways of describing, constructing and measuring the amount of turn, including using geometric instruments. In the early grades, students will experience turn through their own movements. Emphasise that, like other mathematical ideas, practical work is essential if students are to understand how a turn is measured. This includes visualising, using apparatus, and drawing and interpreting diagrams.

Say that angle is a measurement. The angle between two lines is not the shape formed by the two lines, nor the space between them, but is a particular kind of measurement.

Explain that there are two ways to think about what is being measured.

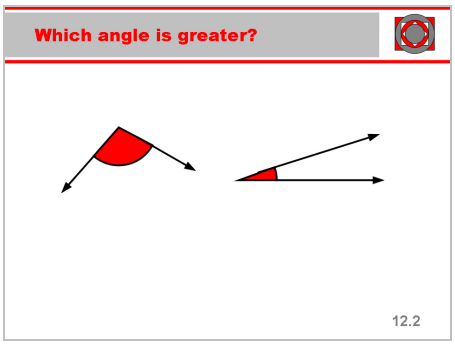
The *dynamic* view is the size of the rotation when you point along one arm of the angle and then turn on the spot to point along the other arm. Explain that this is a good way to introduce children to the concept of angle, because it lends itself to practical experience, with children themselves pointing in one direction and then turning through various angles to point in other directions. On paper, students can physically point something, such as a finger or an arrow, along one arm, and rotate about the intersection of the lines to point along the other.

Use **slide 12.1** to stress that there are always two angles to consider when turning from one direction to the other, the clockwise rotation and the anticlockwise rotation.



Second, there is the *static* view of angle. This is where we think about how pointed is the shape formed by the two lines. We can think of it as a measurement of the difference in direction.

Show **slide 12.2**. Explain that the angle on the left is greater than the angle on the right because in the angle on the right the two arms are pointing in nearly the same direction.



Explain that there are two ways to help students to compare the size of angles:

- *dynamically*, by physically doing the rotations involved, e.g. with an arrow or pencil;
- *statically*, by cutting out one angle and placing it over the top of the other to see which is the most pointed.

The equivalent to making measurements of length, mass and capacity using non-standard units is to measure angles in turns and fractions of a turn, using the dynamic view. For example, a turn from north to east is a quarter turn.

Task 2: Giving instructions

Ask teachers to work in pairs and to turn to **Handout 12.2**. One teacher secretly draws a pathway between points A and B on the paper. The teacher then gives their partner instructions for drawing an identical path.

Teachers then compare the two paths and discuss what difficulties there were and what helped to resolve them.

Now consider whether the task would be different if the instructions were related to the eight points of the compass. Ask pairs to discuss this, then take feedback.

Exploring angles

45 minutes

Say that the next stage is to introduce a standard unit for measuring angle, the degree, where 360 degrees is one whole turn. This unit originated in the Middle East, in ancient Babylon, and is thought to relate to the Babylonian year being 360 days.

The instrument used to measure degrees is the protractor. Students often have difficulty in knowing how to position the protractor along one arm of the angle and imagine the rotation through to the other arm. There are many examples of software that can be used to help students to develop this skill.

Task 3: Using a protractor

Demonstrate one example, such as the interactive teaching program *Angles*, which can be downloaded free from www.standards.dfes.gov.uk/primary/mathematics.

Say that once students have some experience of measuring angles, they should learn to recognise acute, obtuse and reflex angles and to estimate the size of different angles.

Task 4: Estimating angles

Explain that one of the difficulties many students experience is how angles change in size rather than the length of the arms of the angle. The introduction of computer software has greatly helped. There are now a number of different programs that demonstrate and model angles, reinforcing the idea of rotation.

Use the computer program *Angle*. Teachers guess the size of the angle before checking.

Task 5: Angles in a triangle

Ask each group of teachers to mark out a large triangle with masking tape on the floor. One teacher carries out the instructions below as they are read out.

- Stand at one vertex of the triangle ready to walk backwards along one side. Note the direction that you are facing.
- Walk backwards along one side of the triangle.
- Turn through (i.e. over) the internal angle of the triangle.
- Walk forwards along the second side.
- Turn your back through (i.e. over) the internal angle of the triangle.
- Walk backwards along the third side.
- Turn through (i.e. over) the internal angle of the triangle. Note the direction that you are facing.

Ask groups to create a different triangle and to repeat the activity. What do they notice? (When they finish they should be standing in the same spot but facing in the opposite direction from when they started, which indicates they have turned through 180 degrees.)

Show this again on an overhead projector, using an arrow made from card or a short pencil with a point. Explain that this is a dynamic approach to the angles of a triangle.

Compare this with a static view of the same property. Cut out a paper triangle, mark the three angles, tear off the three corners and fit them together to discover that they form a straight line.

Apply the same procedures to a quadrilateral. This time the arrow does a complete rotation, ending up pointing in the same direction as it started. Say that we can conclude that the sum of the four angles of a four sided figure is four right angles.

Show the static view of the same property by tearing off the corners of a paper quadrilateral and fitting them together round a point.

Finish the activity by showing how a quadrilateral can be divided into two triangles. This is another way of seeing that the angles add up to 360 degrees or four right angles.

Task 6: Extension activity

Refer everyone to **Handout 12.3**. Ask teachers to work in small groups to complete the task.

Take feedback.

Show how the results can be proved statically. Choose a vertex of the polygon with n sides and call this point P. Join P to every other vertex of the triangle. This creates $n - 2$ triangles, each of which has an angle sum of 2 right angles. The total of all the angles of the triangles form the sum of the angles of the polygon, which is therefore $2(n - 2)$ right angles.

An alternative proof is to choose a point P inside the polygon and draw lines to each vertex. This divides a polygon with n sides into n triangles. The sum of the angles of these n triangles is $2n$ right angles. To find the sum of the angles of the polygon, we must subtract the n angles around the point P, which together have a sum of four right angles.

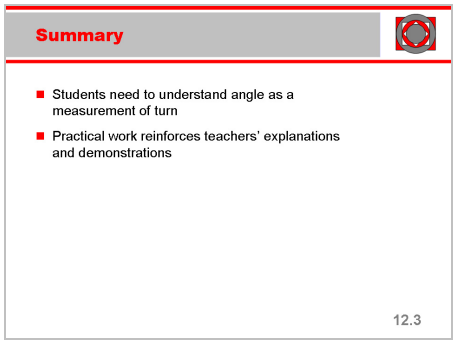
Notes

A five-sided figure has 5 × 2 − 4 right angles.
A six-sided figure has 6 × 2 − 4 right angles.
A seven-sided figure has 7 × 2 − 4 right angles.
An n-sided figure has n × 2 − 4 right angles, or $2(n - 2)$ right angles.

Summary

5 minutes

Refer to **slide 12.3**.



Review the session by summarising the key ideas.