

Fractions, decimals and percentages

Objectives

By the end of this session teachers will:

- have discussed how students learn;
- have seen how visualisation is a key component in teaching fractions;
- have considered some strategies for teaching that are consistent across grades.

Resources

For the trainer

- Computer with data projector, Microsoft PowerPoint and Presentation 8b.ppt
- Whiteboard and flipchart
- Cups, fractions of cups and number cards

For each teacher

- *Teacher's pack*
Handout 8b.1

For each small group

- Cups, fractions of cups and number cards

Session outline

Introduction Slides 8b.1–8b.3	Whole group presentation and discussion	15 minutes
The four operations Handout 8b.1	Whole group presentation Paired work Task 1: Addition and subtraction of fractions Task 2: Multiplication and division of fractions	55 minutes
Equivalence	Whole group presentation Paired work	15 minutes
Ratio and percentages	Whole group presentation Paired work	25 minutes
Summary	Summary	10 minutes

Introduction

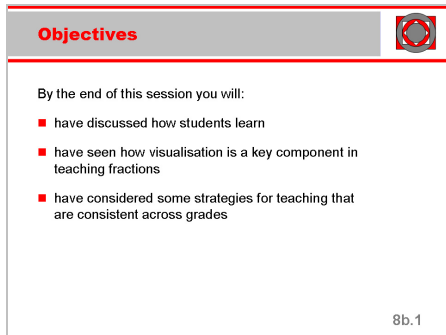
15 minutes

Say that this session will focus on how to teach fractions, decimals and percentages using a consistent approach across grades. It will start by considering how students learn. The need for consistent visual images will be discussed.

Before everyone arrives, brief any interpreter about the key points of the session.

Load **Presentation 8b.ppt**.

Show the objectives for the session on **slide 8b.1**.



Objectives

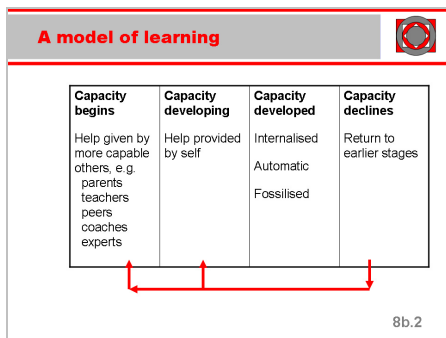
By the end of this session you will:

- have discussed how students learn
- have seen how visualisation is a key component in teaching fractions
- have considered some strategies for teaching that are consistent across grades

8b.1

Explain that you will be covering almost all of the number and algebra standards from Grades 5 to 9. It is essential that students have a sound grasp of these topics because they will need to use and apply them in Grades 10 to 12.

Ask the question: *How do people learn?* Refer to **slide 8b.2**.



A model of learning

Capacity begins	Capacity developing	Capacity developed	Capacity declines
Help given by more capable others, e.g. parents, teachers, peers, coaches, experts	Help provided by self	Internalised Automatic Fossilised	Return to earlier stages

8b.2

Explain that the diagram is a representation of Vygotsky's model of learning. Explain that the model describes how people construct knowledge for themselves with the help of others. Initially they need maximum help from other students, teachers, parents, and so on. As they become more competent, they are able to help themselves. They then internalise that knowledge and use it automatically. If there is a decline, they need to return to earlier stages of learning to refresh and renew the knowledge.

For example, children begin to learn number facts by working out and visualising relationships between numbers. They do this with the help of their parents, teachers and other students. They memorise number facts through repetition and chanting so that they can recall facts without hesitation. Sometimes they forget the facts and they need to go back and remind themselves again.

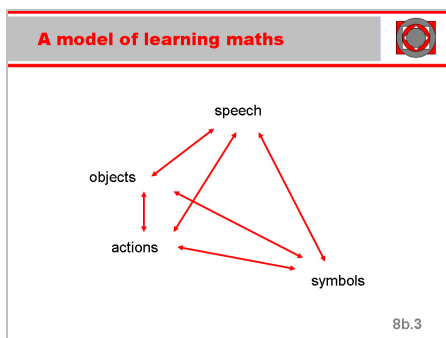
Proficient students categorise and make connections in their developing knowledge so that many things are understood as being of the same type or linked. Recall is made easier by searching for a category and then within a

category. Less proficient students do not instinctively think in categories or make connections but all students need to be taught these skills.

Explain that mathematics consists of a few ‘big ideas’. The big idea of ‘operations’ includes addition, subtraction, multiplication and division. All (real) numbers obey the same laws and conventions. We need to teach operations with numbers in a consistent way so that students can see the links and connections.

Say that when we teach students how to operate with fractions we need to make links and connections with operations with whole numbers. Yet teachers rarely make enough links for their students. Most textbooks have separate chapters on fractions, decimals and percentages, and little effort is made to make the links explicit.

Refer to **slide 8b.3**.



Explain that you will also stress the need to link speech, objects, actions and symbols. Say:

When I show you this one fifth of a cup it provides a visual image. When later I say ‘one fifth’ I want you to be able to visualise this part of a cup. When you look at this part of a cup, I also want you to visualise these symbols and words:

[Write on the flipchart] $\frac{1}{5}$, $\frac{1}{5}$, one fifth, a fifth

And when I write any of those symbols I want you to be able to visualise the part of the cup.

Similarly, when I show you this tenth of a cup I want you to be able to visualise the symbol for the fraction representation, the symbol for the decimal representation and the symbol for the percentage representation. In order for you to be able to do this I need to teach these things explicitly.

Stress that you have not suggested that any visual image will do. The aim is to use a specific object for teaching mathematics and to use this consistently for whole numbers, fractions and decimals. Explain that when we use different objects we are using and applying our knowledge of mathematics. Again, this needs specific teaching so that students can make the links between mathematics and real-life problems.

The four operations

55 minutes

Ask a teacher to help you with your demonstration. Place plenty of cups, fractions of cups, whole number and fraction cards on a resources table. Make sure that the flipchart is in an accessible position.

The following demonstrations should take about 20 minutes.

Use spoken instructions to get your helper to do the following.

- Demonstrate counting whole numbers using cups and cards. Link to symbols on the flipchart.
- Demonstrate addition and subtraction of whole numbers using cups and cards. Link to symbols on the flipchart.
- Demonstrate counting fifths using cups and cards. Link to symbols on the flipchart.
- Demonstrate addition and subtraction of fifths using cups and cards. Link to symbols on the flipchart.
- Demonstrate addition and subtraction of eighty-sevenths using cards.
- Demonstrate addition and subtraction of one hundred and thirty-oneths using cards.
- Demonstrate addition and subtraction of algebraic fractions using cards.
- Demonstrate addition and subtraction of tenths using cups and cards. Link to symbols on the flipchart.
- Demonstrate addition and subtraction of tenths again and link to the decimal representation.

Use written symbolic instructions to get your helper to demonstrate the addition and subtraction of fractions and decimals. Ask the audience to speak the instructions after writing them using symbols.

Task 1: Addition and subtraction of fractions

Refer to **Handout 8b.1**. Get teachers to work in groups. Ask them to take turns to demonstrate the examples. Give them 10 minutes for this activity.

Move on to demonstrate multiplication and division. The following demonstrations will take about 15 minutes.

- Demonstrate multiplication and division of whole numbers using cups and cards. Link to symbols on the flipchart.
- Demonstrate multiplication and division of fifths using cups and cards. Link to symbols on the flipchart.
- Demonstrate multiplication and division of tenths using cups and cards. Link to fraction and decimal symbolisation on the flipchart.

Task 2: Multiplication and division of fractions

Refer again to **Handout 8b.1**. Get teachers to work in groups. Ask them to take turns to demonstrate the examples. Give them 10 minutes for this activity.

Trainers who would like to know more about the 'cups and number cards' approach could refer to: Sue Jennings and Richard Dunne (2003) / *See Maths: Teaching Resources for Books 1, 2 and 3* (Letts Educational; ISBNs 1 84085 6920, 1 84085 6912, 1 84085 6955).

Equivalence

15 minutes

Explain that the demonstrations so far have shown how necessary it is to work with fractions with the same denominator. Students need a very good understanding of the four operations with fractions with the same denominator before moving on to consider fractions of different denominators. Say that we cannot add things of different denomination, making a link to adding money of different denominations and the need for conversion.

Use a cup to demonstrate equivalence. Start with fifths and tenths. Speak the phrase 'same value, different appearance' to reinforce equivalence. Get teachers to imagine or visualise as you pretend to cut an imaginary cup into hundredths and thousandths. Write the equivalence in symbols. Use fraction, decimal and percentage symbolisation.

Explain that once students are confident with the idea of equivalence it is only a small step to repeat the work on addition and subtraction, introducing the additional step to find equivalent fractions.

Demonstrate how to convert fractions to decimals and then to percentages using straightforward cases. Move on to examples such as eighths and sixths. Stress the flexibility needed by students to use different techniques. Students often have difficulty recognising that a ratio, fraction, division, decimal and percentage are all ways of representing the same value. For example:

$$5 : 8, \frac{5}{8}, 5 \div 8, 0.625, 62.5\%$$

all have the same value but a different appearance.

Ratio and percentages

25 minutes

Place three red cups and five blue cups on the table. (Any two colours will do.)

- Compare red to blue: smaller.
- Compare blue to red: bigger.
- Compare red to red: same.
- Compare red to all: smaller.
- Compare all to red: bigger.

Write on the flipchart: 2 : 3 of 12 and $\frac{2}{3}$ of 12.

Explain that 2 : 3 of 12 can also be thought of as: 'every time you see three, replace with two'. Demonstrate this, starting with 12 cups. Move the cups into groups of 3, then replace each group of 3 with a group of 2. The final result is 8 cups.

Write on the flipchart: 17 : 100 of 200, $\frac{17}{100}$ of 200, 17% of 200.

Say:

Every time you see one hundred, replace it with seventeen.

I can see one hundred and another hundred. I replace with seventeen and seventeen, giving the answer thirty-four.

Now think of 17% of 254. Imagine 254 cups. How many hundreds can you see? You can see two and a bit of them. You can see 2.54 batches of a hundred. Replace every one hundred with seventeen. You can see $17 \times 2.54 = 43.18$. So 17% of 254 is 43.18.

Students quickly see that they can do a multiplication and division to find the result. Students who do not make this connection for themselves will need further practice and help in making the connection. They will all need help in finding the most efficient method of calculating percentages especially when using a calculator. This will be dealt with in the next session.

Summary

10 minutes

Review the session by reflecting on:

- how students learn;
- the need to teach all students how to categorise mathematical knowledge;
- the visual images used to help teach the topic;
- how to engage students in active learning activities.