هيئة التعليم

EDUCATION INSTITUTE

Mathematics workshop 3

for teachers of Grades 7 to 12

Teacher's pack: Part 2

Developed for the Education Institute by CfBT

Acknowledgements

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Part 2: Contents

Handouts for Day 3 (Grades 10 to 12)

Handouts for Day 4

Handouts for Day 5

Evaluation forms

Reduced copies of slides

* Handout 12.3 will be given out during Session 12.

Introduction

Aims of the workshop

The purpose of the five-day mathematics workshops is to consider the curriculum standards for mathematics and to discuss the implications for planning the curriculum, teaching, learning and assessment.

The workshops aim to help subject leaders and teachers to:

- become more familiar with the new curriculum standards;
- consider the implications of the standards for planning, teaching and assessment;
- start or refine the planning of a mathematics scheme of work based on the standards, and related lesson plans;
- support colleagues as they implement the standards.

Throughout the workshop, time is allowed for you to study sections of the curriculum standards, to consider points for action arising from the sessions, and to make brief notes.

Many of the sessions in this workshop have different themes, depending on the age range of students in the school.

Workshop programme

Day 1: Developing reasoning

08:00	Registration	
Session 1 08:30–10:00	Mental mathematics	90 minutes
Session 2 10:30–12:00	Proportional reasoning	90 minutes
Session 3 13:00–14:30	Geometric reasoning	90 minutes
Session 4 15:00–16:30	Geometric deduction	90 minutes

Day 2: Algebra and trigonometry

10:45	Registration	
Session 5 11:10–12:30	Generating graphs	80 minutes
Session 6 13:30–14:50	Working with graphs	80 minutes
Session 7 15:10–16:30	Trigonometry	80 minutes

Day 3 (Grades 5 to 9): Number

10:45	Registration	
Session 8b 11:10–12:30 and 13:30–14:10	Fractions, decimals and percentages	120 minutes
Session 9b 14:20–16:20	Calculators	120 minutes

Day 3: Grades 10 to 12: Foundation and advanced mathematics

10:45	Registration	
Session 8c 11:10–12:30	Proof	80 minutes
Session 9c 13:30–14:50	Understanding and using functions	80 minutes
Session 10c 15:10–16:30	Applications of calculus	80 minutes

Day 4: Data handling

10:45	Registration	
Session 11 11:10–12.30	Collecting data	80 minutes
Sessions 12–13 13:30–14:50 15:10–16:30	Working with data	160 minutes

Day 5: Probability and statistics

10:45	Registration	
Session 14 11:10–12:30	Probability	80 minutes
Session 15 13:30–14:50	Statistical inference	80 minutes
Session 16 15:10–16:30	The interactive whiteboard	70 minutes

Standards addressed in Session 1 Handout 1.1

Grade 7: 2.4, 3.3, 4.5, 5.6

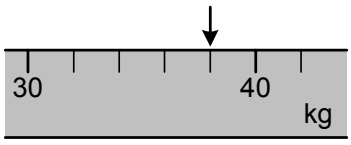
Grade 8: 2.4, 3.1, 6.10, 7.6

Mental tests

Handout 1.2

Write your answers in the boxes below.

1	%

2	
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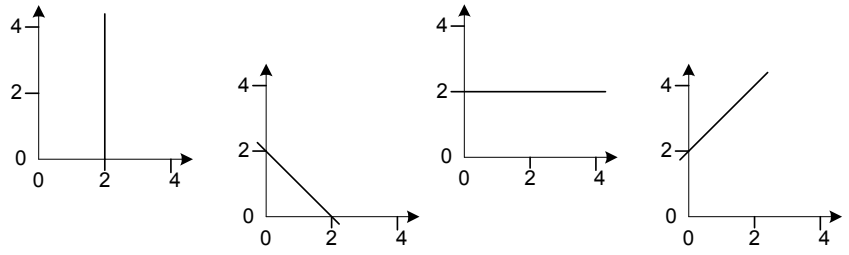
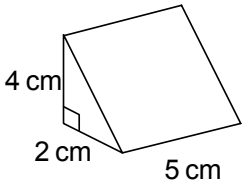
3	mm	4		5	m

6	

7		$m^2(9+m)$

8		9	

10	
	cm^3

Mental tests

Handout 1.3

Test questions

1 What percentage is the same as the fraction one quarter?

2 What number on the scale is the arrow pointing to?

3 How many millimetres are there in nine centimetres?

4 I have a fair six-sided die, numbered one to six. I roll the die. What is the probability that I roll a number less than five?

5 What is the approximate circumference of a circle with a diameter of one metre?

6 Look at the graphs. Draw a ring around the graph that could have the equation y equals x plus two.

7 Multiply out the expression.

8 Write down three consecutive even numbers that add up to thirty.

9 What is the square of three thousand?

10 What is the volume of the prism?

Explicit and implicit information

Handout 1.4

A

Explicit information	Implicit information
$3 \times 0 = 0$	$300 \times 0 = 0$
$3 \times 1 = 3$	$300 \times 1 = 300$
$3 \times 2 = 6$	$300 \times 2 = 600$
$3 \times 3 = 9$	$300 \times 3 = 900$
$3 \times 4 = 12$	$300 \times 4 = 1200$
$3 \times 5 = 15$	$300 \times 5 = 1500$
$3 \times 6 = 18$	$300 \times 6 = 1800$
$3 \times 7 = 21$	$300 \times 7 = 2100$
$3 \times 8 = 24$	$300 \times 8 = 2400$
$3 \times 9 = 27$	$300 \times 9 = 2700$

B

Explicit information	Implicit information
$53 \times 0 = 0$	$5300 \times 0 = 0$
$53 \times 1 = 53$	$5300 \times 1 = 5300$
$53 \times 2 = 106$	$5300 \times 2 = 10\,600$
$53 \times 3 = 159$	$5300 \times 3 = 15\,900$
$53 \times 4 = 212$	$5300 \times 4 = 21\,200$
$53 \times 5 = 265$	$5300 \times 5 = 26\,500$
$53 \times 6 = 318$	$5300 \times 6 = 31\,800$
$53 \times 7 = 371$	$5300 \times 7 = 37\,100$
$53 \times 8 = 424$	$5300 \times 8 = 42\,400$
$53 \times 9 = 477$	$5300 \times 9 = 47\,700$

C

Explicit information

×	20	9	
50	1000	450	1450
3	60	27	87
		Answer:	1537

Standards addressed in Session 2 Handout 2.1

Grade 7:	1.1–1.5, 4.4, 4.5, 4.6, 4.7, 4.8, 5.1, 6.1–6.7, 8.3, 8.4, 8.5, 10.2, 10.4
Grade 8:	1.1–1.7, 3.1, 3.3, 3.4, 4.4, 5.2, 5.3, 5.6, 6.4, 6.9, 7.1, 7.2, 7.4, 7.7
Grade 9:	1.1–1.8, 2.6, 2.7, 2.8, 3.5, 4.1, 4.2, 5.3, 5.5, 5.6, 7.3, 7.4
Grade 10F:	1.1–1.14, 3.6, 3.7, 4.5, 4.9, 5.3, 5.6, 5.7, 5.8, 6.3, 6.5, 7.1, 7.3
Grade 11F:	1.1–1.14, 3.2, 3.3, 4.3, 5.1, 5.4, 5.5, 5.16, 6.3, 7.1, 7.3
Grade 12 F:	1.1–1.14, 3.1, 4.1, 5.2, 7.1, 7.2, 7.3, 9.1, 9.3
Grade 10A:	1.1–1.14, 3.6, 3.7, 4.7, 5.6, 5.7, 5.8, 5.16, 5.17, 6.3, 6.5, 7.1–7.4
Grade 11A:	1.1–1.14, 4.1, 5.10, 5.11, 5.12, 9.1, 9.2, 9.3, 11.1, 11.2
Grade 12A:	1.1–1.15, 5.1

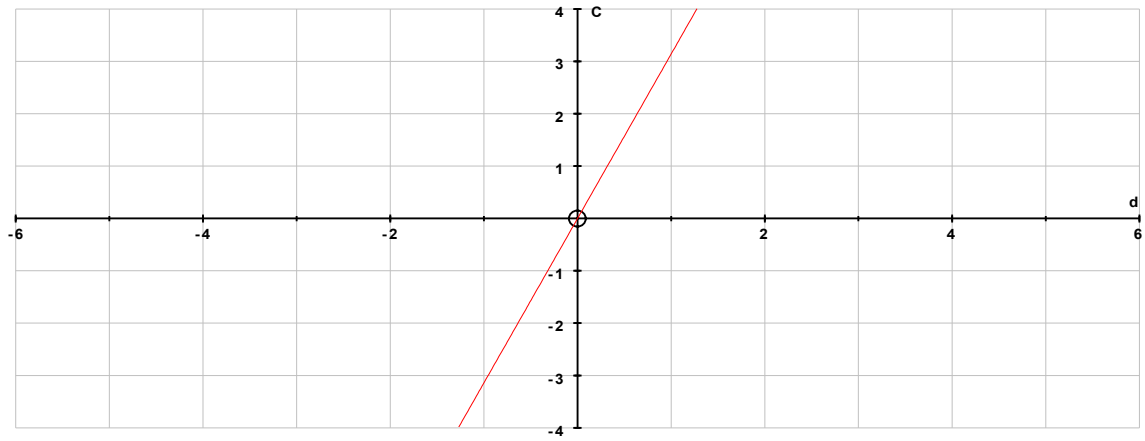
Task 1: Circles

Measure the circumference and diameter of a number of cylindrical objects. Put the measurements in a table.

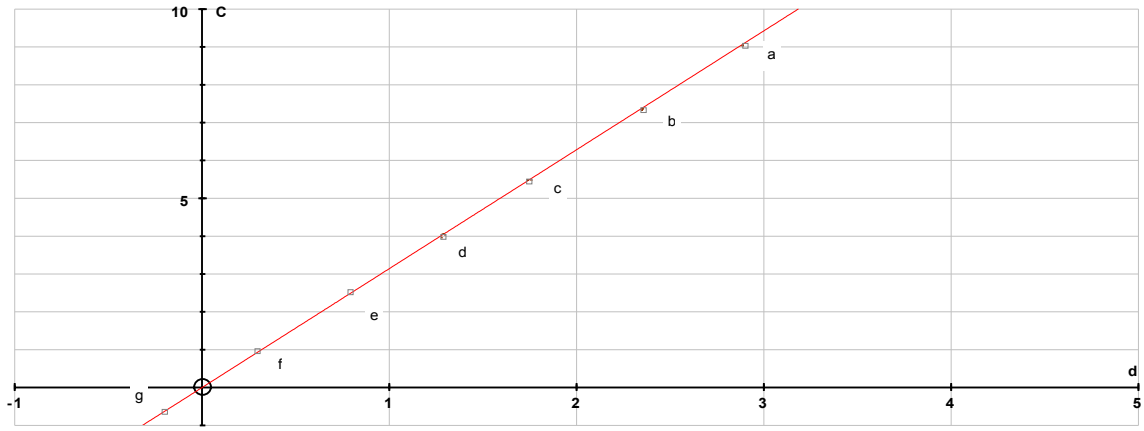
Circumference *C*	Diameter *d*

Draw a graph. Look for the relationship between the circumference and diameter.

Graph to show the relationship between the circumference and diameter of a circle



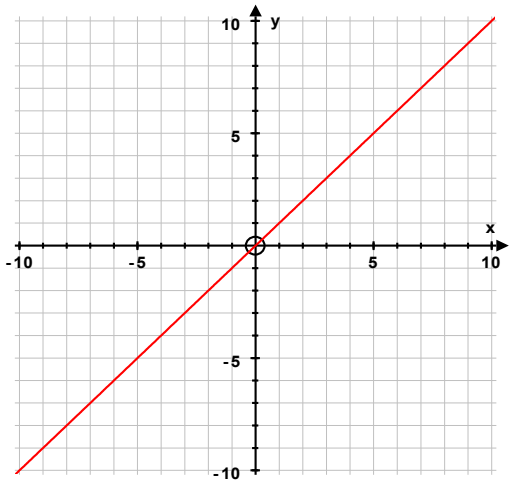
Look at points *a* to *g* on the graph below and describe the circles on which they lie.



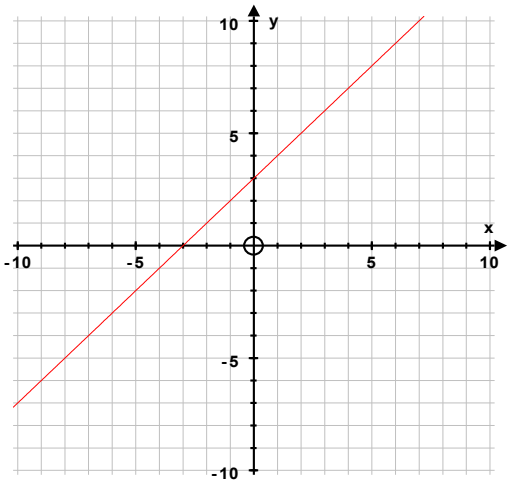
Task 2: Interpreting graphs

Which of the nine graphs show proportional relationships?

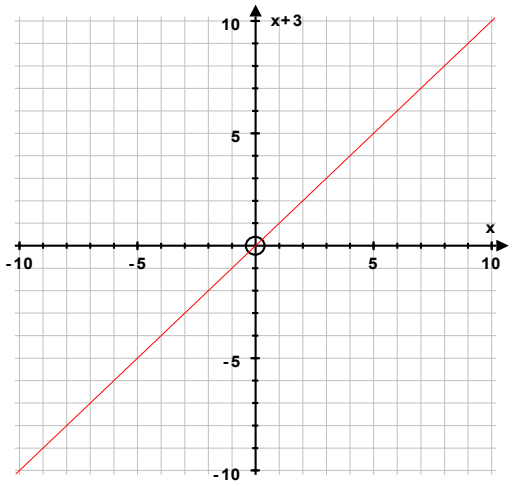
A



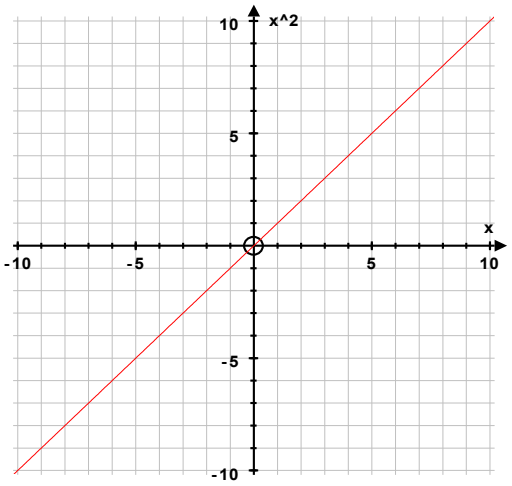
B



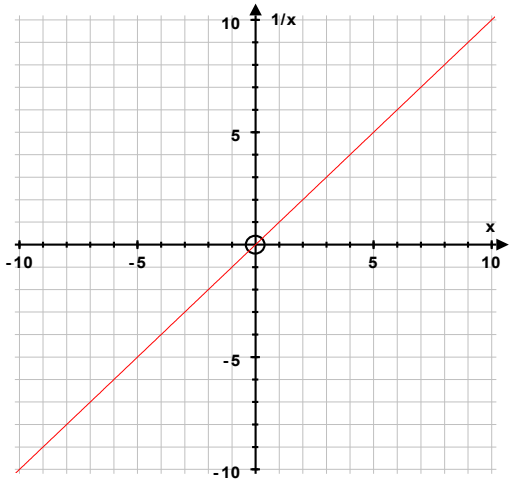
C



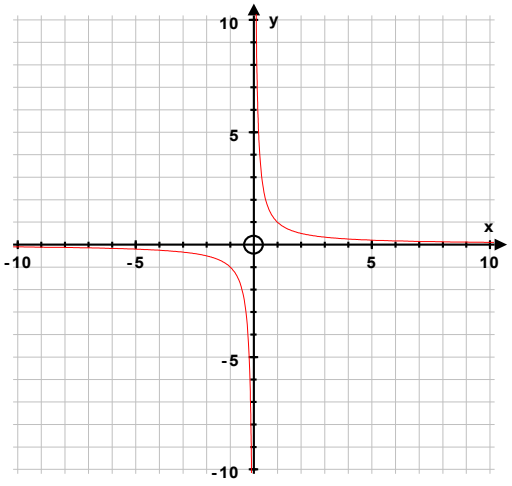
D



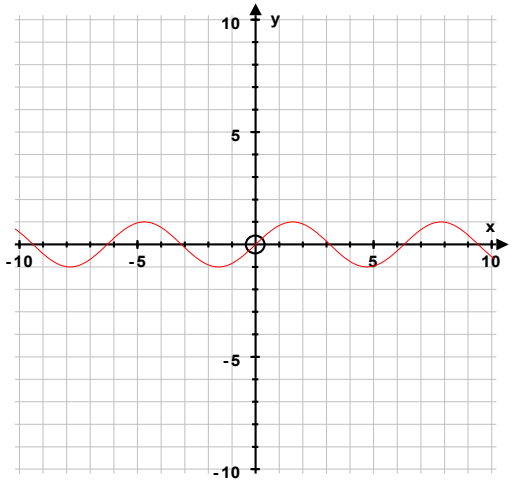
E



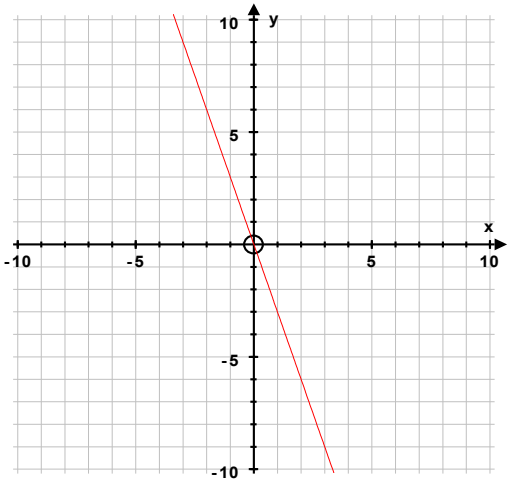
F



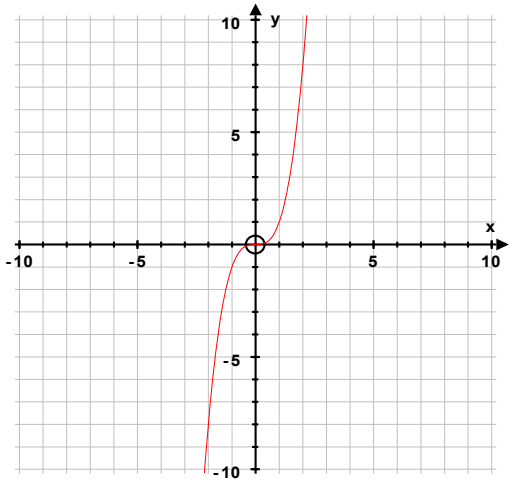
G



H



I



Task 3: Problems

A Look at the following equations. Discuss which variables are proportional.

1 $y = 3(x^2 - 1)$

2 $y = 3x^2 - 1$

3 $y = \dfrac{3}{x^2}$

4 $y = \sin x$

5 $y = 2\tan x + 2$

B The distance, d, moved by an object is proportional to the square of the time, t, for which it has moved. Write down the proportionality relationship and the proportionality formula.

C In each of the tables below, work out whether

$y \propto x$, $y \propto x^2$, $y \propto x^3$ or $y \propto \dfrac{1}{x}$.

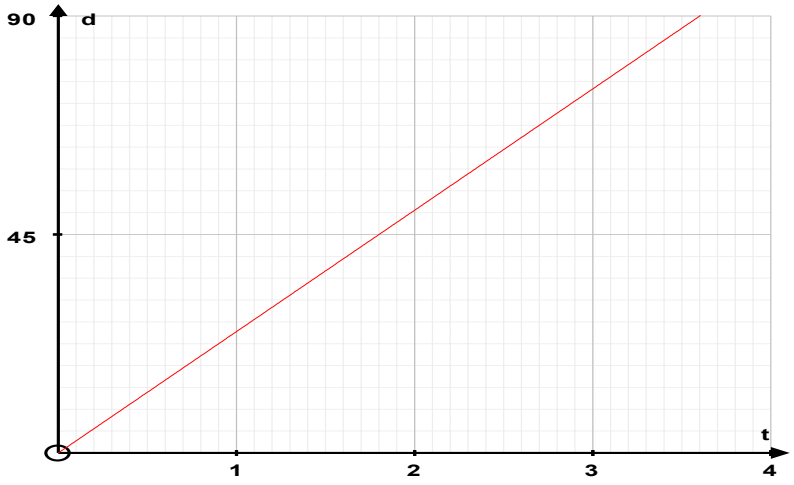
x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	0.5	2	4.5	8	12.5	18	24.5

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2

x	1	2	3	4	5	6	8	12
y	12	6	4	3	2.4	2	1.5	1

x	–2	–1	0	1	2	3	4	5
y	–2	–0.25	0	0.25	2	6.75	16	31.25

D Study the distance–time graph below. Work out the relationship. Write down the equation of the graph.



E Write down the trigonometric ratios for sine, cosine and tangent of an angle θ in a right-angled triangle.

Standards addressed in Session 3 Handout 3.1

Grade 7: 1.4, 9.3, 10.1, 10.2, 10.3

Grade 8: 1.4, 1.5, 7.5

Grade 9: 1.6, 6.1, 6.2

Grade 10F: 1.6, 1.7, 1.11, 7.1

Grade 11F: 1.6, 1.7, 1.11, 7.1

Grade 12 F: 1.6, 1.7, 1.11, 9.1, 9.2

Grade 10A: 1.6, 1.7, 1.11, 7.1, 7.2

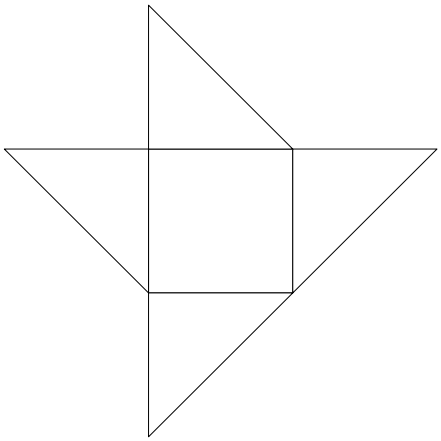
Grade 11A: 1.6, 1.7, 1.11, 11.1

Grade 12A: 1.6, 1.7, 1.11

Geometric reasoning — Handout 3.2

Task 1: Volume of a pyramid

Make three identical pyramids that will fit together to make a cube with an edge length of 5 cm.



Volume of cube =

Volume of pyramid =

The formula for the volume V of a square-based pyramid with base length l and height h is given by:

$V =$

> Archimedes found the volume of a pyramid over 2000 years ago. He had the idea of cutting the pyramid into slices parallel to the base and adding up the volume of the slices. If all the slices are slid towards one corner, we can see that the pyramid has become a third of a cuboid.

Task 2: Surface area of a cone

Cut out a sector of a circle with radius R and angle θ. What is the area of the sector of the circle?

Area of sector =

Use the sector of the circle to make a cone with base radius r. What is the relationship between R and r?

Show that if the slant edge length of the cone is l then the surface area of the cone is πrl.

Task 3: Volume of a cone

Make a cylinder with cross-sectional diameter 12 cm and height 8 cm. Make a cone with the same diameter and height as the cylinder. [Some calculation will be needed.]

Use something like rice to find out how many cones full of rice are needed to fill the cylinder (level at the top).

Volume of cylinder =

Volume of cone =

Formula for volume of a cone with radius of base r and height h =

Task 4: Volume of a sphere

You will need a sphere such as a tennis ball. Measure its diameter.

- Make a cylinder with the same base diameter and height as the sphere and place the ball inside. [This needs to be a tight fit.]
- Make a cone with the same base diameter and height as the cylinder and sphere.
- Fill the cone with rice, level to the top. Pour rice carefully into the space between the sphere and the cylinder at one end. After emptying the rice from that end of the cylinder, turn the cylinder over and fill the other end with the remaining rice. [Allow for experimental error!]

Volume of cylinder =

Volume of cone =

Volume of sphere =

Formula for volume of a sphere with radius r =

> Archimedes was so fascinated by the relationship between the sphere and the cylinder that he requested a suitable diagram to show this to be cut on his headstone.

Task 5: Surface area of a sphere

Read the following argument.

Imagine a sphere made up from many small pyramids. Each pyramid has its apex at the centre of the sphere and has a curved base which is part of the surface of the sphere. The height of each pyramid is approximately the radius of the sphere.

Volume of one pyramid = $\frac{1}{3}$ × curved base area × radius

Volume of all pyramids = $\frac{1}{3}$ × total base area × radius

But this is equal to the volume of the sphere = $\frac{4}{3}\pi \times (\text{radius})^2$

So the total curved base area, which is the surface area of the sphere, must be $4\pi \times (\text{radius})^2$.

Extension questions to do if you complete the tasks quickly

1 Show that the surface area of a sphere is the same as that of a cylinder of the same diameter and height.

2 The diameter of the Earth is about 12 740 km. Work out (a) the approximate volume of the Earth and (b) the approximate surface area of the Earth.

3 The largest monument ever built is the Quetzacoatl pyramid in Mexico. It is 54 metres tall with a square base of edge length 426 metres. Find the volume of the material used to build it (to the nearest 1000 m^3).

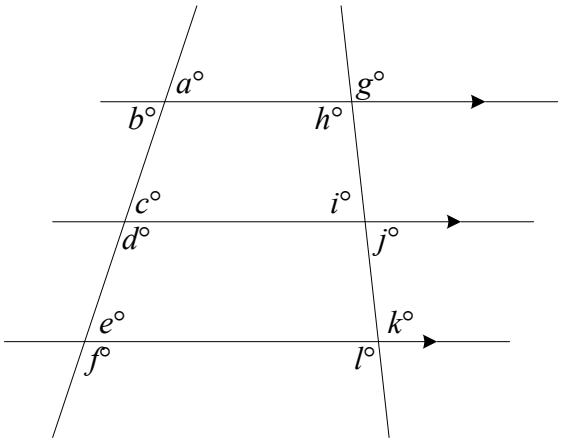
4 The Great Pyramid of Cheops was made of limestone. It had a square base of edge length 230 metres and a height of 146 metres. A cubic metre of limestone weighs 2.268 tonnes. Find the weight of the pyramid (to the nearest 1000 tonnes).

Standards addressed in Session 4 Handout 4.1

Grade 7:	1.4, 9.1, 9.2, 9.5
Grade 8:	1.4, 1.5, 6.1, 6.3, 6.7, 6.8
Grade 9:	1.6, 1.7, 5.1
Grade 10F:	1.5, 1.6, 1.7, 1.8, 6.1, 6.9, 6.11
Grade 11F:	1.6, 1.7, 1.8, 6.1, 6.14
Grade 12 F:	1.6, 1.7, 1.8, 6.1
Grade 10A:	1.6, 1.7, 1.8, 6.1, 6.4, 6.9, 6.11
Grade 11A:	1.6, 1.7, 1.8, 8.1, 8.9
Grade 12AS:	1.6, 1.7, 1.8, 15.1

Task 1: Problems

1



Using the labelled angles above write down:

(a) three pairs of corresponding angles;

(b) three pairs of alternate angles;

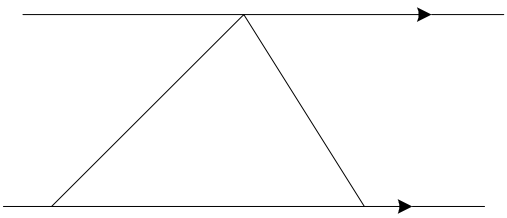
(c) three pairs of vertically opposite angles;

(d) three pairs of supplementary angles.

If $c = 60°$ and $g = 108°$, work out the size of angles a to l.

2 Imagine that you have four sticks of these lengths:
6 cm, 3 cm, 2 cm, 6 cm.
List all the different triangles that you could make.
Name each triangle, using side and angle properties.

3 Use the diagram below to prove that the sum of the interior angles of a triangle is 180°.



Circle theorems

Handout 4.3

1 The angle subtended at the centre of a circle is twice the angle subtended by the same chord at the circumference (in the same segment).

2 Angles subtended at the circumference by the same chord (in the same segment) are equal.

3 Any angle at the circumference subtended by the diameter is a right-angle.

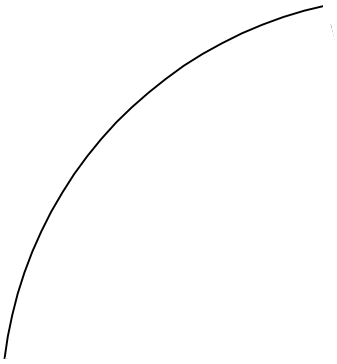
4 The angle between a chord and a tangent is equal to the angle subtended by the chord in the alternate segment.

5 Tangents from a point to a circle are equal.

6 Opposite angles of a cyclic quadrilateral are supplementary.

Task 2: Problems

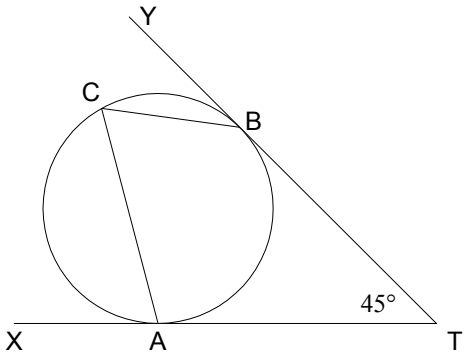
1 Using only a ruler and a pair of compasses, find the approximate centre of the arc of the circle below.



2 ABCDE is a pentagon, with ABCD a cyclic quadrilateral. $\angle ABC = 105°$

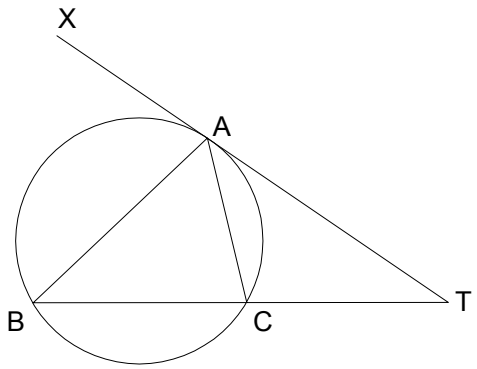
(a) Make a sketch of ABCDE.
(b) Calculate $\angle ADC$.
(c) If $\angle AEC = 75°$, what can you conclude about point E?

3 XAT is a tangent to a circle at A.
YBT is a tangent to the same circle at B.
Calculate $\angle ACB$.



4 TAX is a tangent to the circle at A.
Prove that triangles TAC and TBA are similar.

Hence show that $TC \times TB = TA^2$.



5 Two circles touch each other externally at P.
APX and BPY are straight lines such that A, B lie on one circle and X, Y on the other.
Prove that AB is parallel to XY.

6 Two chords of a circle, AB and CD, intersect at X.
BX = 15 cm
AX = 6 cm
CX = 4 cm
DX = a cm

Find the length of DX.

Standards addressed in Session 5 Handout 5.1

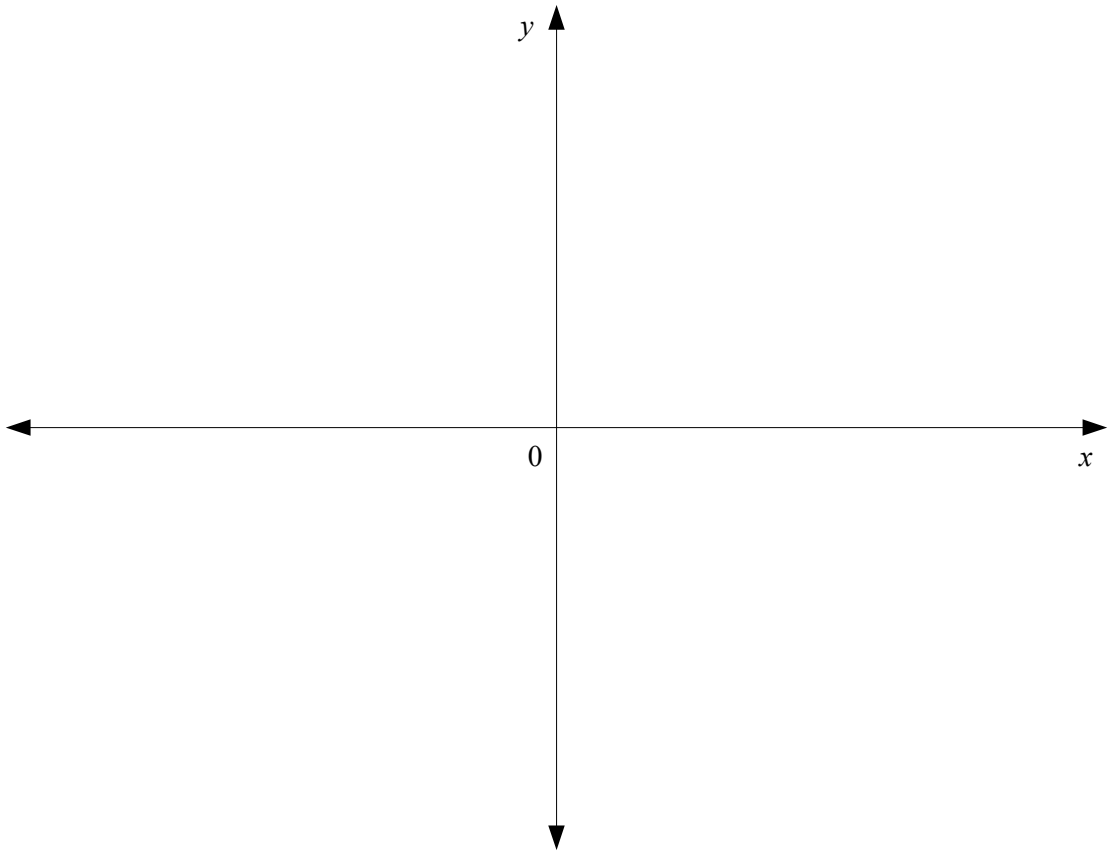
Grade 7:	8.4, 8.5
Grade 8:	1.2, 5.1, 5.4, 5.5, 5.6, 5.7
Grade 9:	1.3, 4.1, 4.2, 4.3, 4.4
Grade 10F:	1.14, 5.1, 5.2–5.15
Grade 11F:	1.14, 5.1–5.10
Grade 12 F:	1.14, 5.1–5.8
Grade 10A:	1.14, 5.1–5.20
Grade 11A:	1.14, 5.1–5.17
Grade 12AS:	1.16, 5.1–5.6

Drawing graphs 1 Handout 5.2

Part A

Draw graphs of the following linear equations on the same set of axes on the computer. Sketch the graphs on the axes below.

$y = x$	$y = x + 1$	$y = x + 2$
$y = x + 3$	$y = x + 0.5$	$y = x + 2.4$
$y = x - 1$	$y = x - 4$	$y = x - 1.5$

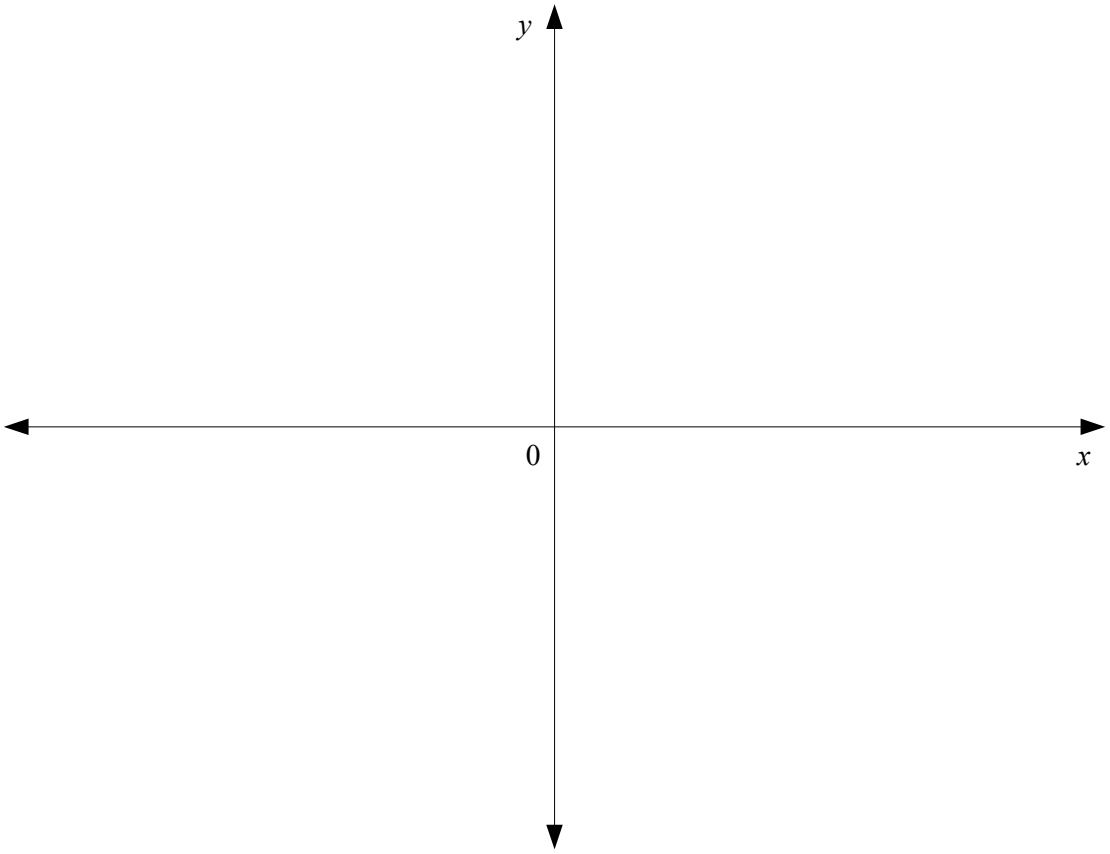


Write down what you notice about these graphs.

Part B

Draw graphs of the following linear equations on the same set of axes on the computer. Sketch the graphs on the axes below.

$y = x$	$y = 2x$	$y = 3x$
$y = 4x$	$y = 5x$	$y = 0.5x$
$y = -x$	$y = -2x$	$y = -0.5x$

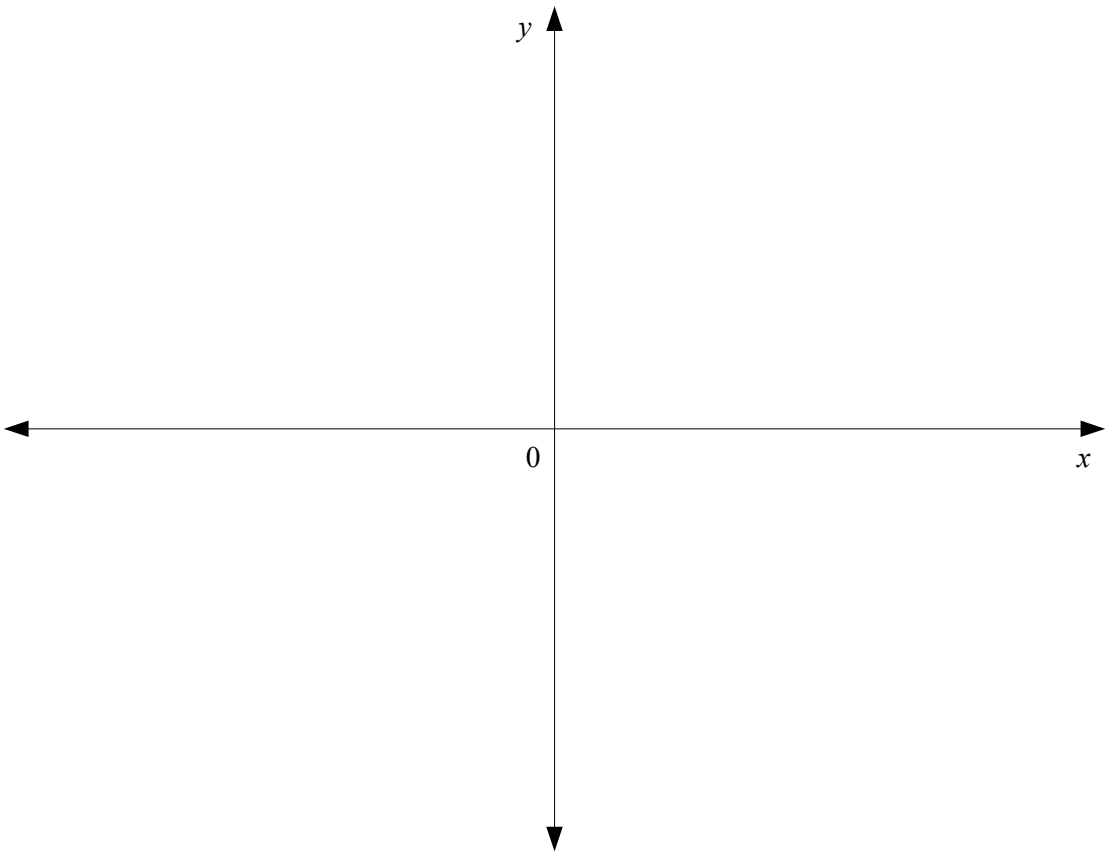


Write down what you notice about these graphs.

Part C

Draw graphs of the following linear equations on the same set of axes on the computer. Sketch the graphs on the axes below.

$y = 2x + 1$	$y = 3x - 3$	$y = 4x - 5$
$y = -5x + 3$	$y = -2x - 0.5$	$y = 7x + 3.5$
$y = 2x - 10$	$y = 0.5x - 4$	$y = -3.5x + 1.5$



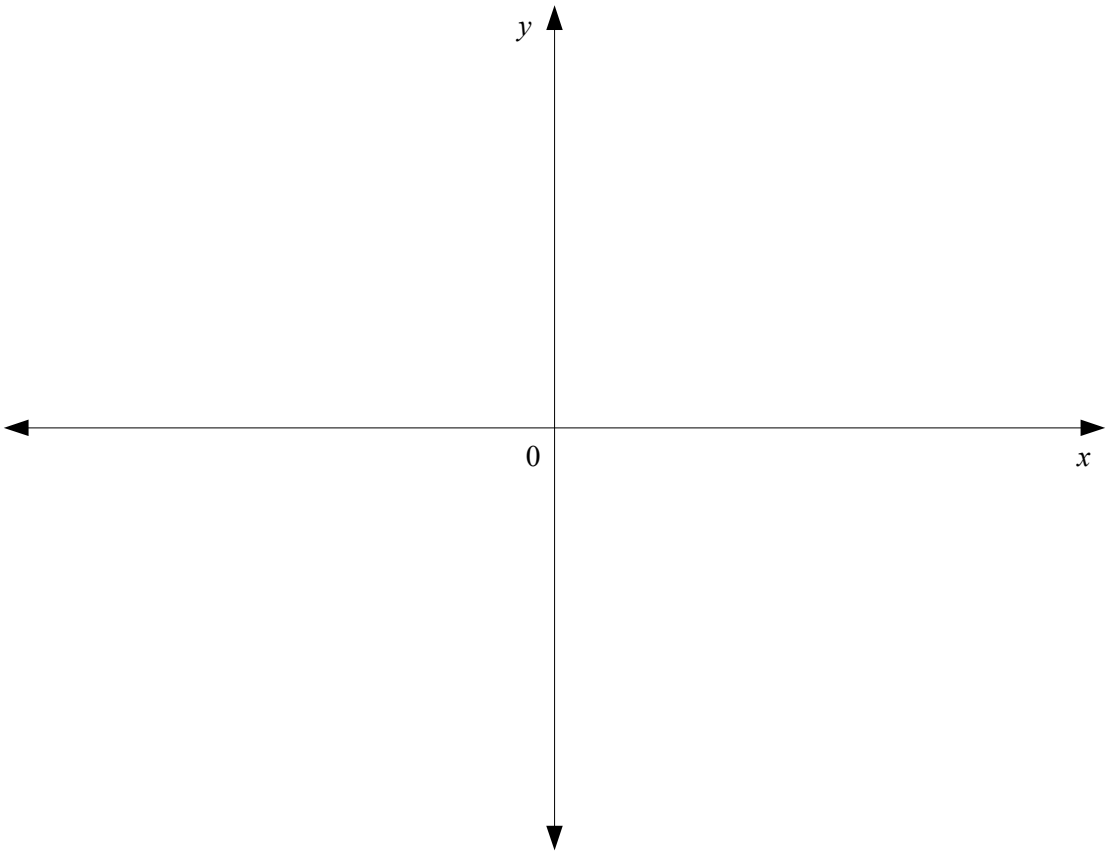
Write down what you notice about these graphs.

Drawing graphs 2 Handout 5.3

Part A

Draw graphs of the following quadratic equations on the same set of axes on the computer. Sketch the graphs on the axes below.

$y = x^2$	$y = x^2 + 1$	$y = x^2 + 2$
$y = x^2 + 3$	$y = x^2 + 0.5$	$y = x^2 + 2.4$
$y = x^2 - 1$	$y = x^2 - 4$	$y = x^2 - 1.5$



Write down what you notice about these graphs.

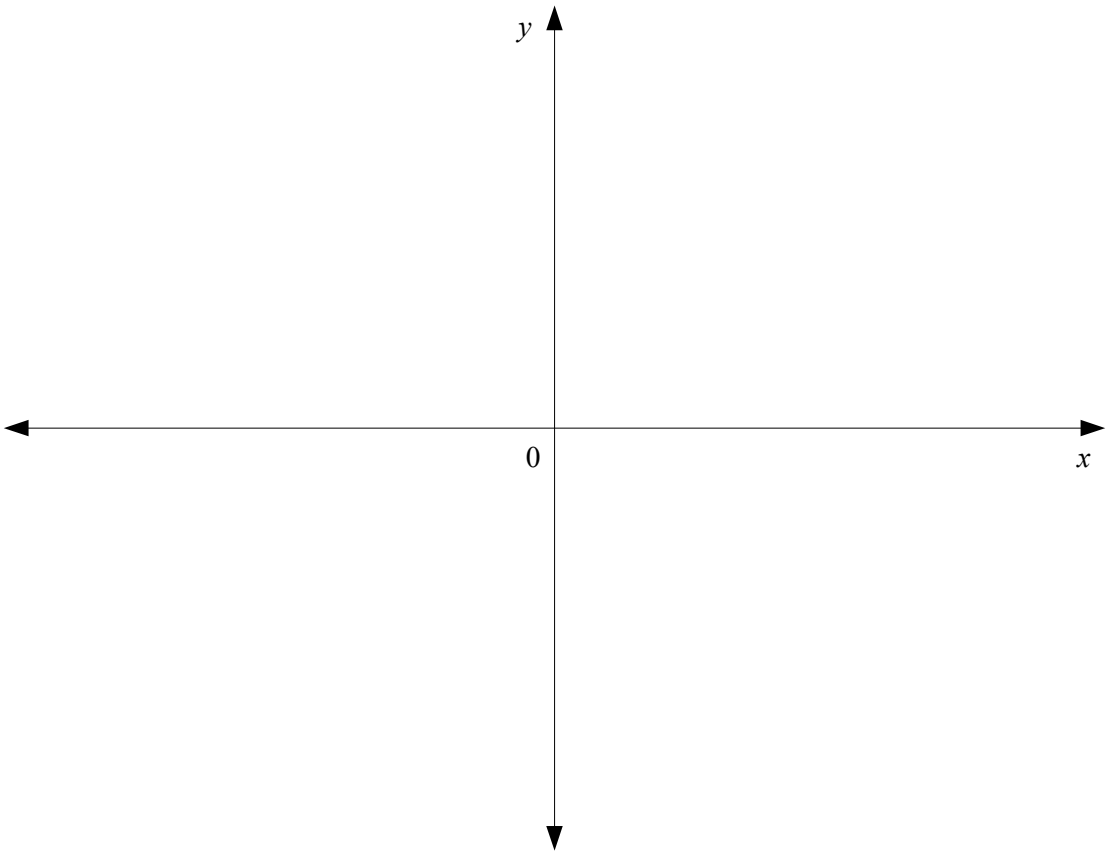
Part B

Draw graphs of the following quadratic equations on the same set of axes on the computer. Sketch the graphs on the axes below.

$y = x^2$ $\quad$ $y = 2x^2$ $\quad$ $y = 3x^2$

$y = 4x^2$ $\quad$ $y = 5x^2$ $\quad$ $y = 0.5x^2$

$y = -x^2$ $\quad$ $y = -2x^2$ $\quad$ $y = -0.5x^2$



Write down what you notice about these graphs.

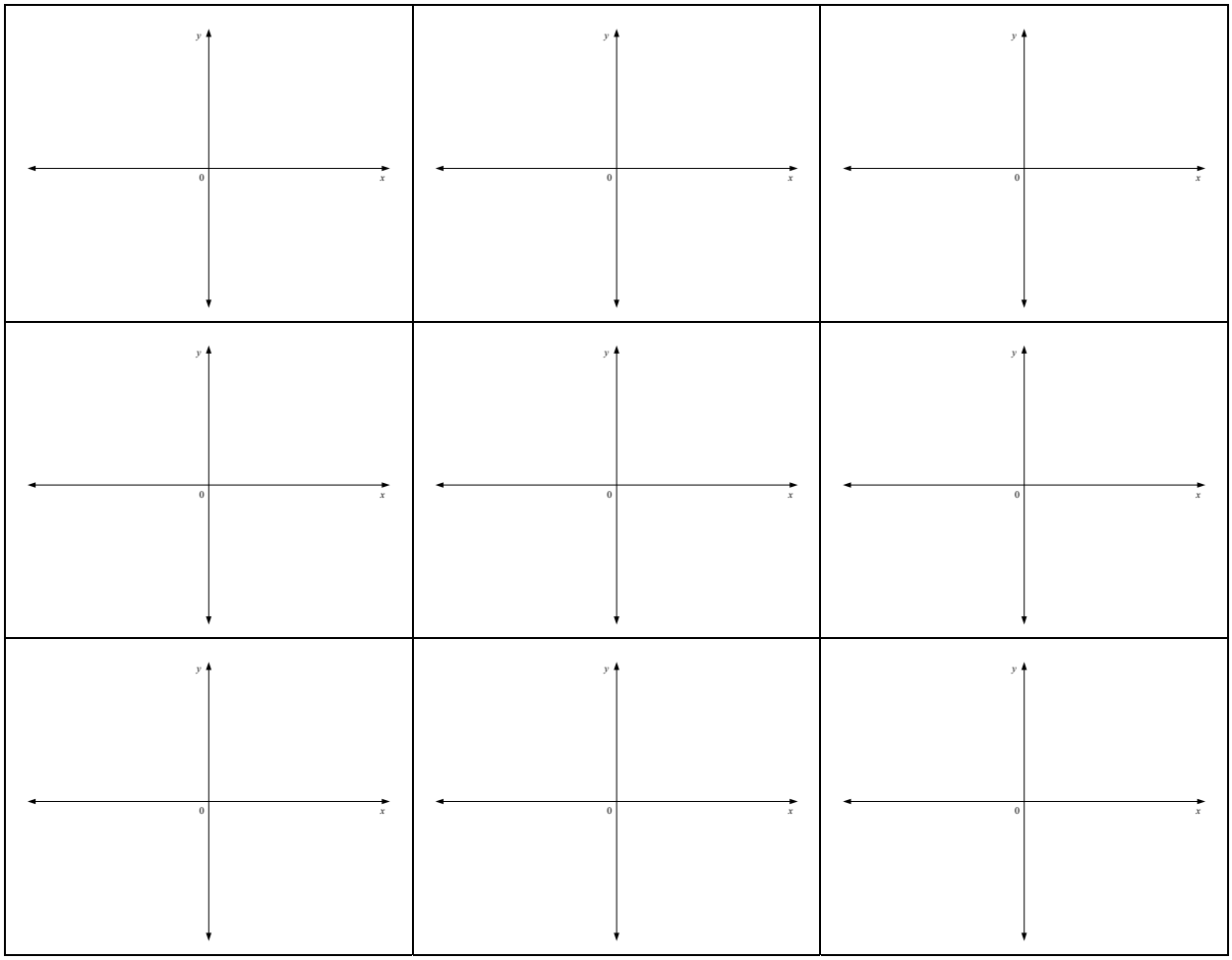
Part C

Draw graphs of the following quadratic equations on different sets of axes on the computer. Sketch the graphs on the axes below.

$y = (x+1)^2$ $\quad y = (x-1)^2$ $\quad y = (x+2)^2$

$y = (x-2)^2$ $\quad y = (x+5)^2$ $\quad y = (x-4)^2$

$y = (x+1)(x-1)$ $\quad y = (x+2)(x-3)$ $\quad y = x(x+1)$



Write down what you notice about these graphs.

Drawing graphs 3 — Handout 5.4

Part A

A cubic equation has the form $y = ax^3 + bx^2 + cx + d$.

Investigate the graphs of these equations by using different values of *a, b* and *c*. You could begin by looking at the following equations. Remember to sketch the graphs as you go.

$y = x^3$	$y = x^3 + 1$	$y = x^3 + 2$
$y = x^3 + 3$	$y = x^3 + 0.5$	$y = x^3 + 2.4$
$y = x^3 - 1$	$y = x^3 - 4$	$y = x^3 - 1.5$

Now investigate further. Take care with your axes or you will not see enough of the graph to see its shape. Write down what you notice about these graphs.

Part B

Now investigate further by looking at the graphs of a general quartic equation:

$$y = ax^4 + bx^3 + cx^2 + dx + e$$

Write down what you notice about these graphs.

Part C

Using the results of your previous investigations make a conjecture about graphs of:

$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ and $y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$

Write down what you notice about the graphs of these polynomial equations.

Standards addressed in Session 6 Handout 6.1

Grade 7:	1.2, 7.4, 7.5
Grade 8:	1.2, 4.7, 4.8
Grade 9:	1.3, 3.6, 3.7, 3.8, 3.9
Grade 10F:	1.14, 4.1, 5.12, 5.13, 5.14
Grade 11F:	1.14, 5.8, 5.11, 5.13, 5.14
Grade 12 F:	1.14, 5.1
Grade 10A:	1.14, 4.1, 4.11, 5.13, 5.14
Grade 11A:	1.14, 5.5, 5.6, 5.7, 5.8, 5.11
Grade 12AS:	1.16, 6.1, 6.2, 6.4

Solving equations 1 Handout 6.2

Task 1: Linear equations

Find the solution of the equation $x+5=9$.

Draw the graphs of $y=x+5$ and $y=9$.

What is the value of x where the two graphs intersect?

Exercise 1

Use graphs to solve the following equations.

1. $x+2=8$
2. $x+7=3$
3. $2x+3=11$
4. $3x-2=13$
5. $2x+3=4x-5$
6. $3x-4=x+12$

Task 2: Simultaneous linear equations

Part A

Find the solution of the simultaneous equations below by drawing the graphs of each equation and finding the point of intersection.

$y=2x+1$

$y=x+3$

Part B

Find the solution of the simultaneous equations below by rearranging the equations in the form $y=ax+b$, drawing the graphs and finding the point of intersection.

$2x+3y=7$

$3x+5y=11$

Exercise 2

Use graphs to solve the following simultaneous equations.

1. $4x+3y=5$ and $3x-2y=8$
2. $5x-2y=4$ and $2x+3y=13$
3. $3x+7y=12$ and $4x-3y=-14$
4. $2x+4y=7$ and $x+3y=5$
5. $3x+5y=1$ and $2x+6y=2$
6. $2x+3y=5$ and $x-4y=19$

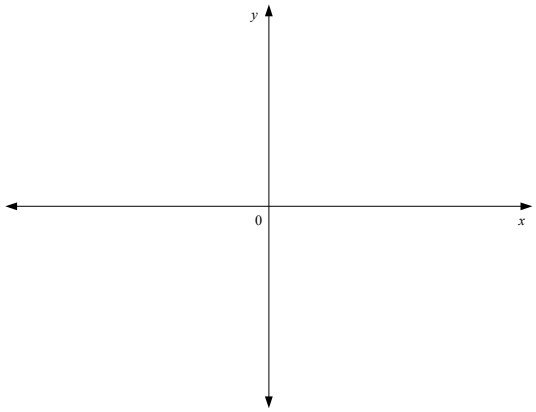
Solving equations 2 Handout 6.3

Task 3: Trial and improvement

Problem

Solve the equation $x^2 + 2x - 6 = 0$.

Use the graph of $y = x^2 + 2x - 6$ to find approximate solutions to one decimal place. Sketch the graph on the axes below. Mark the values of the points where the graph intersects with the x-axis ($y = 0$).



Use a calculator to work out your solutions to five decimal places. Show your working in a table like the one below. The first five lines of one of the solutions have been completed for you.

Approximation	$x^2 + 2x - 6$	Too big	Too small
1.6	−0.24		✓
1.7	0.29	✓	
1.63	−0.0831		✓
1.64	−0.0304		✓
1.65	0.0225	✓	

Exercise 3

Use trial and improvement to solve the following equations.

1. $x^2 + 3x - 5 = 0$
2. $x^2 - 4x + 1 = 0$
3. $x^2 + x - 4 = 0$
4. $2x^2 + 3x - 1 = 0$
5. $-x^2 - 2x + 1 = 0$

Task 4: Factorisation

Part A

Draw the graph of $y=(x-1)(x+2)$

Look at the intersection of the graph with the *x*-axis. This is where *y*=0.

What is the solution of $(x-1)(x+2)=0$?

What is the solution of $x^2+x-2=0$?

Part B

Draw the graph of $y=x^2-2x-3$

Factorise the right-hand side of the equation.

Now write down the solution of $x^2-2x-3=0$

Exercise 4

Use factorisation to solve the following equations. Check your answers by drawing appropriate graphs.

1. $x^2+x-6=0$
2. $x^2-x-12=0$
3. $x^2-5x+6=0$
4. $x^2+5x+6=0$
5. $x^2-4x+4=0$

Task 5: Extension

Exercise 5

Estimate the solutions to the following equations using their graphs.

1. $x^3-3x+1=0$
2. $x^3-7x+7=0$
3. $x^3-4x-1=0$
4. $x^4-3x^2-4x-3=0$
5. $x^3+2x^2+1=0$

Task 6: Completing the square

Look again at the equation

$$x^2 + 2x - 6 = 0\,.$$

Can the equation be rearranged in such a way that we have an equation like this?

$$(x + p)^2 = q^2$$

Yes.

$$x^2 + 2x - 6 \equiv (x+1)^2 - 7$$

Use the graphics calculator or computer to check that the two graphs

$$y = x^2 + 2x - 6 \quad \text{and} \quad y = (x+1)^2 - 7$$

are identical. You can see that the two graphs are coincident. The points where one graph crosses the x-axis are identical to the other.

Now consider the solution of

$$\begin{aligned}(x+1)^2 - 7 &= 0\\ (x+1)^2 &= 7\\ (x+1) &= \pm\sqrt{7}\\ x &= -1 \pm \sqrt{7}\end{aligned}$$

Now consider the general quadratic equation

$$ax^2 + bx + c = 0$$

Divide by a $\quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Rearrange $\quad \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} = \frac{b^2}{4a^2}$

Simplify $\quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$$x + \frac{b}{2a} = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use this general rule to solve the equations in Exercise 3 and to check your answers.

Standards addressed in Session 7 Handout 7.1

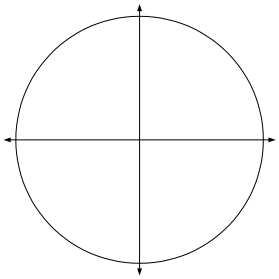
Grade 7:	1.2, 9.1
Grade 8:	1.2, 5.4, 5.5,
Grade 9:	1.3, 1.7, 4.4, 5.5, 7.3, 7.4
Grade 10F:	1.1–1.14, 5.4, 6.3, 6.5
Grade 11F:	1.1–1.14, 5.1, 6.8, 6.9, 6.10, 6.11, 6.12
Grade 12F:	1.1–1.14, 5.2
Grade 10A:	1.1–1.14, 5.4, 6.3, 6.5, 6.6
Grade 11A:	1.1–1.14, 5.12, 8.2, 8.6
Grade 12AS:	1.1–1.16

Defining sine and cosine

Handout 7.2

Task 1: The unit circle

On a sheet of plain paper, draw a unit circle, radius 1 unit (10 cm).



Mark points on the circumference of the circle at every 10° and draw the radii.

For each point on the circumference, measure the coordinates (x, y) to two decimal places. (How many measurements do you actually need to make?)

Each x-value is called the **cosine of the angle** and each y-value is called the **sine of the angle**. Record your measurements in the table below.

Check your measurements using your calculator.

Use your measurements to draw the graphs of cosine θ and sine θ on the same axes.

Angle θ in degrees	Cosine θ	Sine θ	Angle θ in degrees	Cosine θ	Sine θ
0			180		
10			190		
20			200		
30			210		
40			220		
50			230		
60			240		
70			250		
80			260		
90			270		
100			280		
110			290		
120			300		
130			310		
140			320		
150			330		
160			340		
170			350		
180			360		

Extension 1

Consider a point P on the circumference of the unit circle. What construction do you have to make find the value of the tangent at that point?

Extension 2

Use graphical software to explore the transformations of the trigonometric functions. For example:

$y = \sin\theta + a$

$y = a\sin\theta$

$y = -\sin\theta$

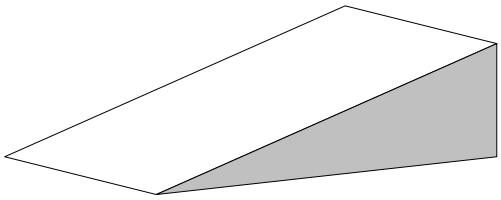
$y = \sin -\theta$

Task 2

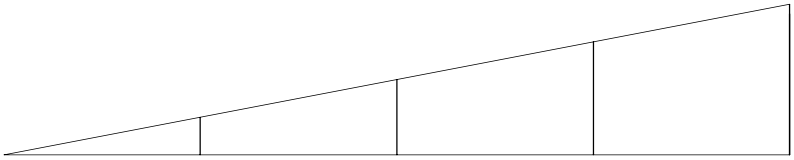
Exercise 1

1 A 10 m ladder is placed against a wall. The greatest angle of inclination it can reach before slipping is 75°. What is the highest point it can be placed at on the wall to the nearest 0.1 m?

2 A wooden skateboarding ramp is being made. The ramp will be 1.5 m above the ground at its highest point and the slope of the ramp must be 35°. What length of wood is needed to make the sloping face of the ramp to the nearest 0.01 m?



3 A road is built on four pillars, each 25 m apart. Calculate the height of each pillar. The angle of inclination of the road is 6°.



4 In the Louvre in Paris there is a square-based pyramid with perpendicular height 21 m. The angle between the slant edge and the square base is 41°. Work out the length of the side of the square base.

5 T is a place on the Earth lying on the line of latitude 62° N. The radius of the Earth is approximately 6400 km. Calculate the radius of the line of latitude 62° N.

6 ABCDEFGH is a cube of side 2 cm.
(a) Calculate the length of the diagonal of one of the faces of the cube.
(b) Calculate the length of the diagonal of the cube.

7 For any triangle ABC, show that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Operations with fractions

Handout 8b.1

Task 1: Addition and subtraction of fractions

Use the cups, fractions of cups and number cards to do the actions that go with the following operations. As you do the actions, get your partner to speak and write the symbols.

1. $\frac{2}{5}+\frac{3}{5}$ 2. $\frac{1}{5}+\frac{4}{5}$ 3. $\frac{3}{10}+\frac{1}{10}$ 4. $\frac{4}{10}+\frac{3}{10}$

5. $\frac{4}{5}-\frac{2}{5}$ 6. $\frac{7}{5}-\frac{3}{5}$ 7. $\frac{7}{10}-\frac{4}{10}$ 8. $\frac{9}{10}-\frac{5}{10}$

9. $\frac{1}{5}+\frac{7}{5}+\frac{4}{5}-\frac{3}{5}$ 10. $\frac{6}{5}+\frac{2}{5}-\frac{4}{5}$

Task 2: Multiplication and division of fractions

Use the cups, fractions of cups and number cards to do the actions that go with the following operations. As you do the actions, get your partner to speak and write the symbols.

1. $\frac{2}{5}\times 3$ 2. $\frac{1}{5}\times 2$ 3. $\frac{3}{10}\times 5$

4. $\frac{7}{10}\times 4$ 5. $\frac{53}{100}\times 3$ 6. $\frac{41}{100}\times 5$

7. $\frac{8}{5}\div\frac{2}{5}$ 8. $\frac{12}{5}\div\frac{6}{5}$ 9. $\frac{9}{10}\div\frac{3}{10}$

Progression in calculator skills — Handout 9b.1

To ensure good continuity in students' learning, you need to be aware of the progression in calculator skills from Grade 5 through to Grade 9.

Students in Grades 5 and 6 are expected to learn to use a basic calculator.

By the end of Grade 6, students should be able to:

- **use a calculator to perform a one-step problem and interpret the result;**

  Most students will have little difficulty with entering a one-step calculation such as 4.5 × 27. However, when they are solving word problems, when it is not always obvious which values and what operations to use, they may misinterpret the question and enter the wrong calculation.

- **key in and interpret money and measurement calculations;**

  Interpreting the results of money calculations often causes difficulties.

- **recognise rounding errors – for example, recognise 2.9999999 as 3;**

  Students need to make sense of problems and realise when answers are likely to have been rounded.

- **use division to enter a fraction such as ⅜, recognising the display of 0.375 as the decimal equivalent;**

- **recognise recurring decimals, such as 0.3333333, and know that this is equivalent to ⅓;**

  Students need to recognise that not all digits may recur in a decimal, as in 1 ÷ 6 = 0.16666666.

- **recognise negative numbers and use the sign-change key if appropriate;**

  Students may miss the minus sign that indicates a negative number, which is usually on the extreme left of the display.

- **carry out calculations with more than one step, such as 8 × (37 + 58), or ⅜ of 980;**

  Students need to be familiar with the order of operations so that they select the correct sequence of operations in calculations involving more than one step.

- **clear the display before starting a calculation;**

  Students are less likely to make errors if they clear the display before starting a new calculation.

- **correct a wrong entry by using the CLEAR ENTRY key.**

  Most students will clear the display and repeat the calculation if they think that they made an error. They also need to learn how and when to use the CE key.

- **recognise the likely size of the answer and check answers appropriately, for example, by carrying out the inverse operation.**

  This is an important skill – errors in making entries often lead to answers that are nonsense.

These skills need to be consolidated, reinforced and built upon throughout the later grades. In summary, by the end of Grade 8 most students should know:

- the order in which to use the keys for calculations involving more than one step;
- how to carry out calculations involving percentages without using the percentage key;
- how to enter numbers and interpret the display when the numbers represent money, metric measurements, units of time or fractions;
- how to carry out calculations involving mixed units of time (e.g. hours and minutes, or minutes and seconds);
- when and how to use facilities such as the memory, brackets, the square-root and cube-root keys, the sign-change key, the fraction key and the constant facility;
- how to select from the display the number of figures appropriate to the context of a calculation.

Making connections

Handout 9b.2

As you solve these problems, consider:

- how you make use of a calculator;
- which standard relates most closely to the problem (the grade and the standard).

---

1 Study the statements below. Say whether they are true or false.

a. $\frac{3}{4}$ has the same value as 75%.

b. $\frac{2}{3}$ has the same value as 60%.

c. 0.33 has the same value as $\frac{1}{3}$.

d. 0.625 has the same value as $5 \div 8$, which has the same value as $\frac{5}{8}$.

2 Which of these numbers is divisible by 9?

A. 225

B. 270

C. 2214

D. 182 734

What conclusion can you draw?

3 Is this statement always true, sometimes true or never true?

'The sum of four even numbers is divisible by 4.'

4 A teacher asked her class to work out $3 \times 5 + 3 - 2 \times 7 + 1$, without using brackets.

She received these answers: 11, 8, 5, 23.

Which of the answers is correct? Explain your answer.

5 Write each of these fractions as a percentage.

a. $\frac{3}{8}$

b. $\frac{7}{12}$

c. $\frac{4}{5}$

d. $\frac{5}{9}$

e. $\frac{5}{7}$

6 Write each of these lists of numbers in ascending order.

a. $\frac{4}{5}$, $\frac{1}{2}$, 0.7, $\frac{1}{10}$, 75%

b. 0.3, 10%, $\frac{1}{4}$, 68%, $\frac{3}{8}$

c. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{4}{5}$

7 Work out the mean (average) of each of the following sets of numbers.

a. 5, 6, 6, 7, 8, 8, 9, 10

b. 2.5, 3.2, 4.4, 4.7, 4.9, 5.3, 6.1

c. 14.2, 14.3, 14.3, 14.4, 14.5, 14.6, 14.7, 14.7, 14.7, 14.8, 14.8, 14.9

8 For each of these statements, say whether it is true or false.

a. $34 \times 79 = 79 \times 34$

b. $169 \div 13 = 13 \div 169$

c. $57 + 61 + 43 + 39 = 61 + 57 + 39 + 43$

d. $94 - 45 + 76 - 32 = 45 - 94 + 32 - 76$

e. $36 \div 5 \times 4 = 36 \div 4 \times 5$

f. $60 \div 5 \div 3 = 60 \div 3 \div 5$

Word problems

Handout 9b.3

For each problem:

- work out what has to be done;
- write down the operation;
- write down the calculator keys you need to press.

---

1 Work out the area of a triangle with base 13 cm and height 4.5 cm.

2 At the start of my journey there were 29.5 litres of fuel in the tank of my car. I used 17.8 litres. How much fuel was left?

3 A recipe for jam uses 56 g of fruit for every 100 g of jam.
A jam jar contains 454 g of jam.
How much fruit is needed to make 15 jars of jam?

4 During one week in January 2002, prices in Argentina rose by 3% each day.
What was the percentage increase in prices after five days?

5 Which of these statements is true?

A. My savings increase by 7% and then decrease by 7%.
The result will be an overall increase.

B. My savings increase by 7% and then decrease by 7%.
The result will be an overall decrease.

C. My savings increase by 7% and then decrease by 7%.
The result will be that my savings stay the same.

6 A student scored 17 out of 45 in a test. Write this score as a percentage.

7 Complete the following sentences.

a. To increase a quantity by 22%, I multiply by …

b. To decrease a quantity by 15%, I multiply by …

8 Sherifa walks 0.74 km to school and the same distance home each day.
How far does she walk in 67 days?

9 A wire fence has a total length of 234 m.
Wooden poles are placed at every 1.5 metres along the length of the wire.
How many wooden poles are needed?

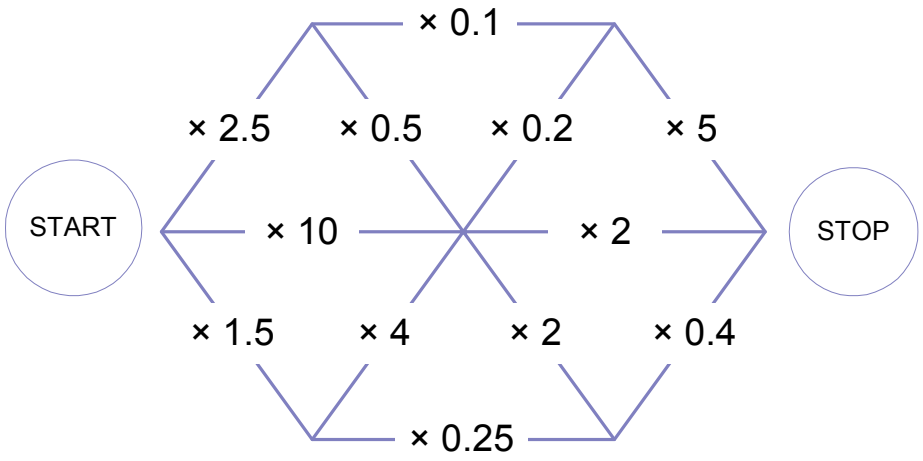
10 The number of students in a school increased from 1800 to 2160.
What was the percentage increase?

Using a calculator to investigate

Handout 9b.4

1 MAZE

Try this puzzle. Start with 1 in your calculator display.



Choose a route from START to STOP.
You may go along each line in only once.
Multiply the number in your display by the number on the line.
The aim is to finish with 5 in your display.

2 MISSING OPERATIONS

Each box represents a missing operation.
Find out what it is.

(37 □ 21) □ 223 = 1000

(756 □ 18) □ 29 = 1218

27 □ (36 □ 18) = 675

41 □ (87 □ 19) = 2788

3 ADDING DIGITS

Use all the digits 1, 2, 3, 4, 5, 6, 7, 8 in order.
Write an expression with a total of 90.

For example: $1 + 23 + 45 + 6 + 7 + 8 = 90$

You can put digits together to make numbers greater than 9 (e.g. 45), or insert operation signs.

Find different ways to do it.

4 REVERSE AND SUBTRACT

Choose any two digits. Repeat them to make a four-digit number, e.g. 3737.
Reverse the number to make another number, e.g. 7373.
Subtract the smaller from the larger, e.g. 7373 – 3737 = 3636.
Reverse the answer, e.g. 6363.
Do another subtraction, e.g. 6363 – 3636 = 2727.
Keep going until you get a three-digit number,
e.g. 7272 – 2727 = 4545, 5454 – 4545 = 909.
Then stop.

Try this with different starting numbers. What happens? Why?

5 THINK OF A NUMBER

Think of a number.
Multiply by 11.
Multiply the answer by 13.
Multiply that answer by 7.

Repeat with different start numbers. What do you notice? Explain your result.

6 LOOKING FOR PATTERNS

$1 \times 9 + 2$
$12 \times 9 + 3$
$123 \times 9 + 4$
Check that the pattern continues, and use it to write down the answer to
$12345678 \times 9 + 9$

7 FOUR FOURS

Use the four digits 4, 4, 4, 4.
You can use any combination of these with the operations +, –, ×, ÷ and brackets.

For example, $4 + 4 + 4 + 4 = 16$, $44 – (4 \div 4) = 43$

Can you make all the numbers from 1 to 100?
Show how you work out each number.

8 USING BRACKETS

Find all the possible values of this expression by inserting brackets.

$11 + 4 \times 9 – 5$

9 PATTERNS WITH FRACTIONS

Write these fractions as decimals: $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, $\frac{4}{11}$, …

Write what you notice.

Investigate other fractions written as decimals such as thirds, fifths, sevenths, ninths.

1 Given that the angles on a straight line sum to 180°, prove that the angles in a triangle sum to 180°.

2 Prove that the base angles of an isosceles triangle are equal.

3 Prove that the only regular polygons that tessellate the plane have 3, 4 or 6 sides.

4 Prove that the product of two numbers is even if and only if at least one of the numbers is even.

5 Prove that if five consecutive integers are squared, then the mean of their squares is 2 more than their median value.

6 Prove that a triangle with sides $2n$, $n^2 + 1$ and $n^2 - 1$ is a right-angled triangle. Is the converse true?

7 Prove that the statement ***every number of the form* 6*n* ± 1 *is prime*** is false.

8 State, with reasons, whether the following statements are true or false.

(a) $x^2 + x + 41$ is prime for all x.

(b) The sum of two prime numbers is a prime number.

(c) $x^2 + y^2 \geq 2xy$

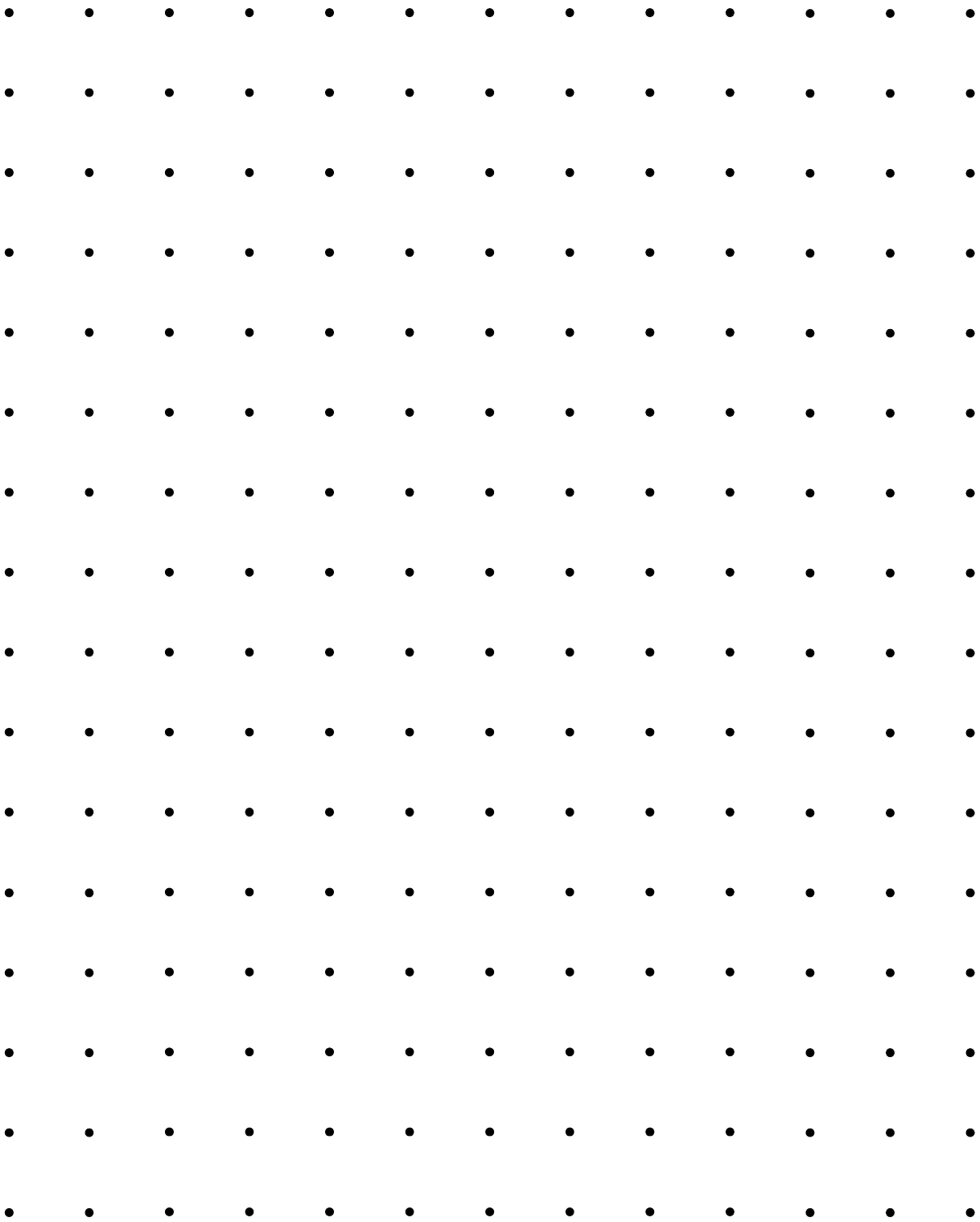
(d) $p + q = 1 \Rightarrow p^2 + q = q^2 + p$

(e) $x^2 + x \leq 9x - 17$

9 Prove that the sum of the first n positive odd numbers is n^2.

Dotty paper

Handout 8c.2



Understanding and using functions Handout 9c.1

Why functions?

Describing functions

What is a function?

Different representations for functions

Some families of functions

Ways of categorising functions

Some important mathematical features of functions

Generating functions

Composite functions

Combinations of functions

Functions defined in piecewise manner

Functions from physical contexts

Some pedagogical issues

Applications of calculus Handout 10c.1

Optimisation

1 A manufacturer wants to make sealed aluminium cans to contain 0.75 litres of liquid. What dimensions should the cans have to minimise the amount of aluminium the manufacturer needs to use for each can?

2 Your windpipe contracts when you cough. The speed v with which wind is ejected from the windpipe is given approximately by the formula $v = k(R - r)r^2$, where k is a constant, R is the normal radius of the windpipe and r is the contracted radius. What value of r maximises the speed with which air is ejected from the windpipe in a cough?

3 Two straight corridors intersect at right angles to each other. One corridor is 2 m wide and the other is 3 m wide. What is the length of the longest ladder that can be carried horizontally around the corner where the corridors meet?

4 A rectangular enclosure has its sides parallel to the north–south and east–west directions. It is 500 m long in the north–south direction and is surrounded by roads, which have a better walking surface than the enclosure. A man walks from a point S on one side of the enclosure to a point P on the on the other side. The 'horizontal' displacement of P from S is 1 km due west, and the 'vertical' displacement is 500 m due north. The man can walk at 2 m s^{-1} on the road, but only at 1 m s^{-1} across the enclosure. What route should the man follow to minimise his walking time?

Modelling

5 The fundamental law of mechanics is Newton's second law of motion, which states that the rate of change of momentum of a body is equal to the total force acting on the body. Momentum is mass times velocity.

A body of mass 1 kg is initially at rest at a point 1 m from another point O. The particle can move along a straight path through O. A variable force of $2x^{-3}$ newtons, in a direction away from O, is then applied to the particle, where x is the distance of the particle from O at time t. Find x in terms of t.

7 Newton's law of cooling states that the temperature of a hot object decreases at a rate proportional to the temperature difference over the surrounding temperature. Rewrite this statement as a differential equation.

8 When a person dies, the normal body temperature of 37°C decreases, eventually to the temperature of the surroundings. Assume that when the surrounding temperature is 20°C it takes 2 hours for the temperature of the corpse to be 35°C.

The police discover a dead body. Its body temperature is 30°C and the temperature of the surroundings is 20°C. The body was found at 15:00 hours. When did the person die?

Collecting data

Handout 11.1

1 Medical records may contain the following information:

- gender (male/female)
- age (years)
- weight (kilograms)
- height (metres)
- ethnic group (Arab, European, African-American, African, …)
- smoker (yes/no)
- systolic and diastolic blood pressures (millimetres of mercury)
- level of calcium in the blood (micrograms per millilitre).

Which of these variables are categorical variables and which are quantitative variables?

2 A teacher keeps records on her students. They look like this:

Student	Main subject	Total marks	Final grade
Haya	Arabic	450	A
Wafa	Mathematics	430	A
Noor	English	400	B
Mona	Science	380	C

Which are the statistical units and which are the variables?
Classify each of the variables as categorical or quantitative.

3 You want to measure students' leisure time and their fitness.
How might you define leisure time?
How might you measure leisure time?
What variables could you use to measure fitness?
How might these be measured?

4 Your students are applying to universities around the world. Suggest five variables that they might consider in order to compare these universities. Give reasons for your choices.

5 Suggest how you might simulate the totals on the repeated throws of two six-sided dice.

6 You want to study the number of deaths from cancer in 2004 compared with 1994. In planning the design of your investigation, is it sufficient merely to look at the number of deaths in each of these two years? Explain your reasoning.

7 What is the best way to find out about the following?

(a) How satisfied students are with the teaching in your school

(b) How student-learning with face-to-face teaching compares with computer-based instruction

(c) The length of time that teachers wait for students to respond to one of their questions before answering it themselves

Writing frame: Setting up an enquiry Handout 11.2

1 Planning an enquiry	What am I going to investigate? Are there any instructions to follow? Is the population specified? Is data supplied? What might we want to know about the population? **Set out a plan.**
2 Writing a hypothesis	How can I frame the question about the population? Are there any assumptions I can make? What might I expect to be the result of my enquiry? Can I give reasons for my hypothesis? **Write a hypothesis. Give explanations and reasons.**
3 Collecting data	Do I need to collect the data? What is the population? Shall I collect data from the whole population or shall I select a sample? Do I need to write a questionnaire? Shall I set up a spreadsheet in which to enter the data? **Explain how the data is being collected.**
4 Selecting data from a given set	Is data provided? How shall I select from the given data? Is the data discrete or continuous? Is it bivariate data? **Explain how to select data and give its source.**
5 Sampling	Is it appropriate to take a sample? How can I ensure that the sample is representative? Shall I do a pilot study to test my sampling techniques? Shall I take (i) a random sample? (ii) a stratified sample? (iii) a quota sample? What size sample shall I take? How can I ensure that my sample is not biased? E**xplain and justify sampling methods.**

6 Statistical techniques	Mode, median, mean, standard deviation – will these calculations help me answer my question? I must select the most appropriate for the data and explain why. Range, interquartile range or percentiles – will any of these help me answer my question? I must explain. Standardising scores? Is this necessary and will it improve my answer? Correlation techniques? Probability? **Decide what techniques to use.**
7 Representing discrete data	I must select the most appropriate representation and explain why I am using it. Frequency table? Bar chart? Pictogram? Pie chart? Stem and leaf diagram? Compound bar chart? **Explain and give reasons for choice of representation.**
8 Representing continuous or grouped data	Frequency table? Grouped frequency table? Frequency diagram? Frequency polygon? Histogram? Cumulative frequency diagram? Box and whisker diagram? Normal curve? **Explain and give reasons for choice of representation.**
9 Representing bivariate data	Scatter graph? Line of best fit? **Explain and give reasons for choice of representation.**
10 Interpreting and communicating results	The diagrams and results need to be explained in terms of my original question. What do the numerical answers tell me? What do the diagrams or graphs tell me? Have I answered the question? Was my hypothesis correct? Could my result be generalised? What have been the limitations of my study? Has there been any bias? Could I do more to improve my study?

Standard deviation Handout 12.1

1 Construct two samples, sample C and sample D, with the same standard deviation but different means.

2 Construct two samples, sample E and sample F, with the same means but with one standard deviation twice the other's standard deviation

3 Calculate the mean and standard deviation of this data set:
$X = \{4, 5, 5, 6, 6, 6, 7, 8, 9\}$
Each member of the set is transformed using $y = 2x + 3$.
Write down the new data set, its mean and standard deviation.

4 Compare the two test results for the same group of 30 students.

Science test results

56	56	57	57	57	58	58	59	60	60
60	62	62	62	62	62	62	63	63	64
64	64	65	66	66	66	66	66	67	68

Mathematics test results

12	15	18	20	22	34	44	50	54	58
62	62	64	66	68	68	70	70	72	75
80	84	86	87	88	90	92	96	98	98

One student had marks of 66 for science and 70 for mathematics. Comment on her results.

Analysing data

Handout 12.2

Data set 1

The first hundred sentences in Charles Darwin's *Origin of the Species* contain the following number of words:

52	73	25	44	15	24	37	27	33	53
34	21	22	38	72	18	95	37	58	38
12	37	63	20	92	49	14	46	47	21
69	38	48	20	13	16	26	13	33	23
44	32	18	53	22	10	22	17	28	83
22	22	28	80	34	36	41	56	16	64
36	21	22	30	25	26	83	28	65	49
41	41	70	56	45	19	19	32	30	36
77	41	27	46	30	64	34	53	38	64
39	11	51	32	39	33	24	21	31	18

What sort of data is this?

What can you do with this data?

Data set 2

The first hundred sentences in James Watson's *Microbiology of the Gene* contain the following number of words:

10	17	18	17	31	43	27	34	40	50
15	20	52	29	41	24	21	18	30	30
14	16	15	12	15	15	17	25	15	12
27	29	27	18	22	18	18	22	23	25
17	22	27	28	9	33	22	19	26	8
32	13	13	33	23	27	22	39	21	27
19	27	21	28	30	30	28	21	28	9
17	29	21	17	21	40	40	43	14	21
21	15	31	21	26	33	35	22	19	16
10	21	27	22	18	46	22	5	17	12

Handout 12.3 will be given out during the session.

A participant on a TV game show has to choose between three closed doors. Behind two of the doors there is a goat, and behind the other door there is a Toyota car. The player has to choose a door.

A player who chooses the door with the car behind it, wins the car. A player who chooses a door with a goat behind it, wins a goat.

The player indicates a door. Before it is opened, the host of the TV show, who knows what is behind each door, opens one of the two remaining doors, which he knows has a goat behind it. He then tells the player that there is one final chance for a change of mind.

Should the player choose a different door?

Probability

Handout 14.2

Some vocabulary of probability

event	success
failure	sample space
independent	dependent
favourable outcome	combinatorics
mutually exclusive	exhaustive
experimental probability	theoretical probability
conditional probability	compound event
distribution	binomial distribution
Poisson distribution	geometric distribution
uniform distribution	normal distribution
expectation	variance
chance	likelihood

Task 1: Complete the sentences

Fill in the missing words to make meaningful sentences.

1 …… events cannot happen at the same time.

2 Events that do not affect each other are called …… events.

3 The …… is the set of all possible outcomes of an event.

4 The …… of the successful outcome of an event is the ratio of the probability of successful outcomes to the probability of failure.

5 In a binomial trial, if p is the probability of success then …… is the probability of failure.

6 …… is the study of different possible arrangements of objects.

7 The probability of an event given that another event has occurred is called a …… probability.

8 In the probability distribution of a discrete random variable the sum of the probabilities of all the possible outcomes for the values of the variable is equal to …… .

9 The probability of an event that is …… to happen is 1.

10 The sum of all possible products of the value of a discrete random variable with the corresponding probability is called the …… of that variable.

Task 2: Probability calculations

1 A bag contains 4 red balls, 3 green balls and 5 blue balls. The balls are all the same size and weight. What is the probability of drawing a red or a blue ball in a random selection?

2 A regular dice is tossed 100 times. How many sixes might be expected?

3 Two regular six-sided dice are rolled. What is the probability of scoring a total score of more than 7?

4 A fair coin is tossed five times. Calculate the probability of scoring at least three heads.

5 A fair coin is tossed five times. Calculate the probability of scoring at least four heads in a row.

6 What is the probability that, when two regular dice are rolled, the first dice shows a 2 or the total on the two dice is 6 or 7?

7 A circular target board has concentric circles of radii 1 cm, 2 cm, 3 cm and 4 cm. A person shoots at the target and scores 40, 30, 15 and 5 if the bullet lands in each of these circles respectively. What is the probability of scoring more than 15 on a single shot, assuming the shot hits the target?

8 A bank-card has a four-digit Personal Identification Number (PIN). Suppose PINs are assigned at random and that each of the digits is equally likely. How many possible PINs are there? What is the probability that the PIN has at least one zero?

Statistical inference — Handout 15.1

Task 1: Confidence intervals

Use the information in the table to help you answer questions 1 and 2.

No. of standard deviations from mean	1.645	1.960	2.576
Confidence interval	90%	95%	99%

1 The level of potassium in a person's blood is subject to minor daily variation. It is known that Aida's potassium levels vary normally with $\sigma = 0.2$.

When her potassium level was measured on one particular day it was 3.4. Give a 90% confidence interval for her mean potassium level.

At another time, Aida's potassium levels were measured over three successive days and gave a mean value of 3.4. What is now the 90% confidence interval for her mean potassium level?

How many measurements of Aida's potassium levels would be needed to estimate her mean potassium level with margin of error $\pm$ 0.06, with a 95% confidence interval?

2 A standard weight of 10 grams is weighed repeatedly to check the accuracy of a weighing device. The device gives readings that are normally distributed with a known standard deviation of 0.0002 grams.

When weighed five times, the mean weight is 10.0023 grams. Give a 99% confidence interval for the mean of repeated measurements of the weight.

At the same level of confidence, calculate how many measurements would be needed to give a margin of error of $\pm$ 0.0001 for the mean value of the weight.

Task 2: Test of significance

3 Slide 15.11 shows a problem related to the blood pressure of a sample of teachers.

How high would the mean systolic blood pressure of these teachers have to be to reject the null hypothesis:

(a) at the 5% significance level?

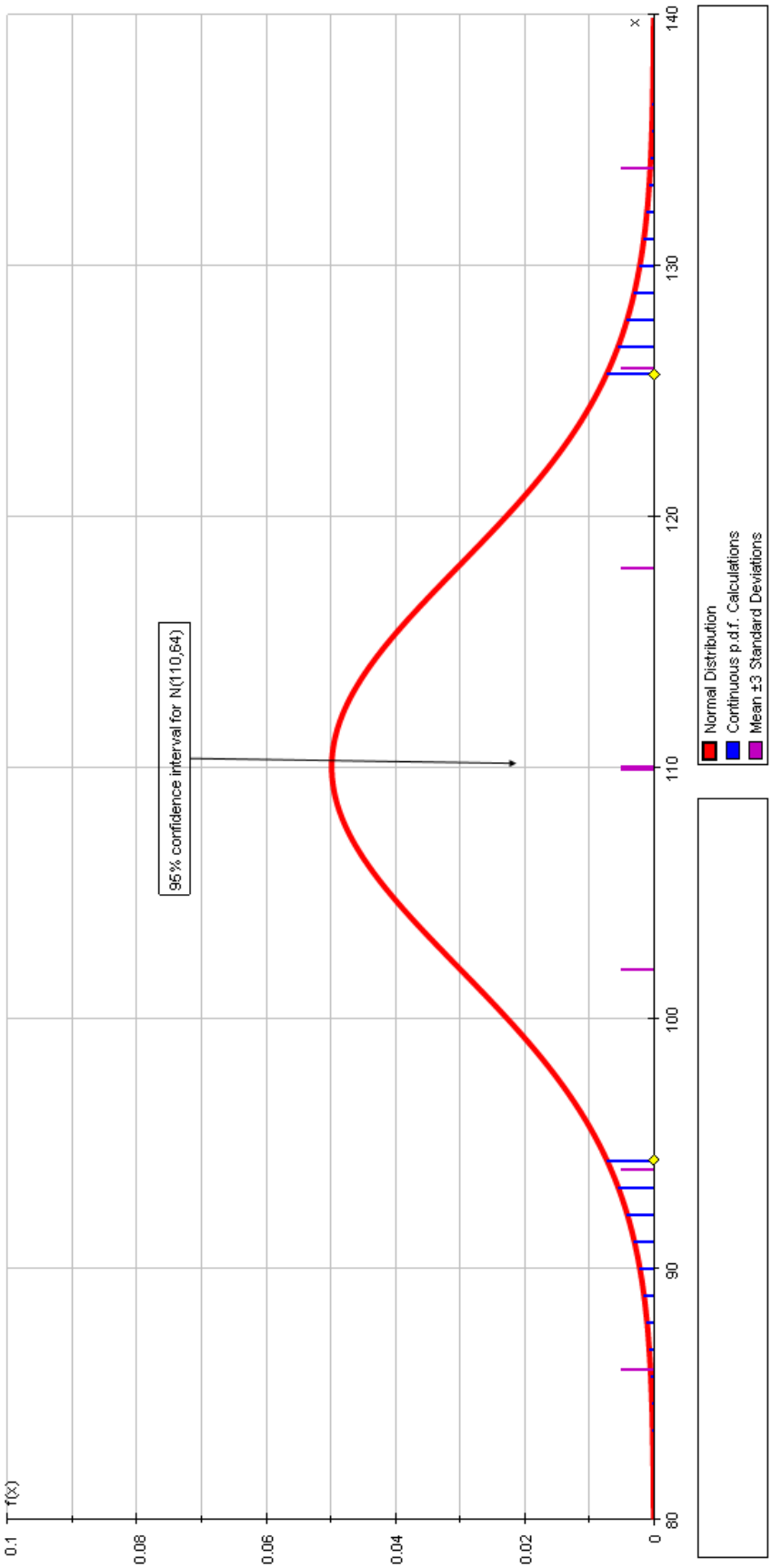
(b) at the 1% significance level?

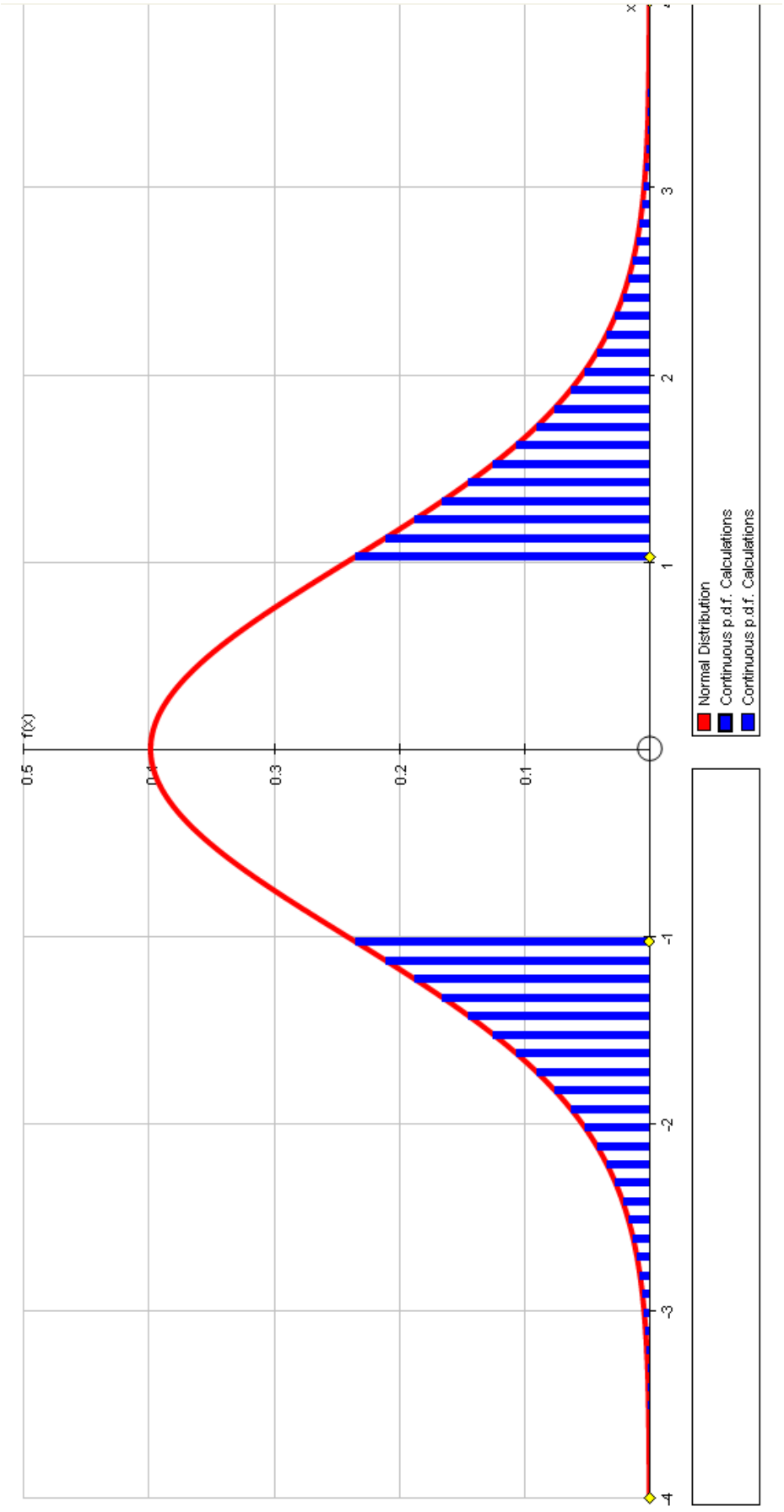
How would the problem change if the alternative hypothesis were the following?

The mean blood pressure of middle-aged male teachers is greater than 130.

95% confidence interval

Handout 15.2





Gap tasks

Handout 16.1

Before the next workshop, aim to complete these tasks.

1 Finish your discussion of the action points to follow up the workshop. You may like to involve your School Support Organisation in this task.

Discuss relevant points with your school principal, and what would help to maximise the benefits to other teachers.

2 Tell other colleagues what you learned on the workshop and discuss with them the implications for what you need to do as a subject team.

3 Plan and teach a series of lessons using ICT, including computers and calculators.

4 Conduct some teaching sessions to focus on mental mathematics.

5 Use some teaching methods involving visualisation.

Teacher's evaluation form: Workshop 3 Day 1

For completion by teachers by the end of Day 1

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session	Grade: please ring					
Mental mathematics	Very good	A	B	C	D	Very poor
Proportional reasoning	Very good	A	B	C	D	Very poor
Geometric reasoning	Very good	A	B	C	D	Very poor
Geometric deduction	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 Day 2

For completion by teachers by the end of Day 2

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session	Grade: please ring					
Generating graphs	Very good	A	B	C	D	Very poor
Working with graphs	Very good	A	B	C	D	Very poor
Trigonometry	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 Day 3 (Grades 5–9)

For completion by teachers of Grades 5 to 9 by the end of Day 3

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session (GRADES 5–9)	Grade: please ring					
Fractions, decimals and percentages	Very good	A	B	C	D	Very poor
Calculators	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 Day 3 (Grades 10–12)

For completion by teachers of Grades 10 to 12 by the end of Day 3

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session (GRADES 10–12)	Grade: please ring					
Proof	Very good	A	B	C	D	Very poor
Understanding and using functions	Very good	A	B	C	D	Very poor
Applications of calculus	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 Day 4

For completion by teachers by the end of Day 4

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session	Grade: please ring					
Collecting data	Very good	A	B	C	D	Very poor
Working with data (2 sessions)	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 Day 5

For completion by teachers by the end of Day 5

What were the best aspects of today's sessions?

What changes would you suggest if today's sessions were repeated?

Please grade each session according to how useful it was.

Session	Grade: please ring					
Probability	Very good	A	B	C	D	Very poor
Statistical inference	Very good	A	B	C	D	Very poor
The interactive whiteboard	Very good	A	B	C	D	Very poor
Overall grade for the day	Very good	A	B	C	D	Very poor

Further comment (optional)

School: ……. Name: ……..

Please return this form to your trainer before leaving.

Teacher's evaluation form: Workshop 3 as a whole

For completion by teachers by the end of Day 5

Please grade the workshop overall according to how useful it was.

Grade: please ring					
Very good	A	B	C	D	Very poor

What was most helpful?

What changes would you suggest?

What sessions would you like to have in the third workshop?

School: ……. Name: ……..

Please return this form to your trainer before leaving.