## Day 1 Session 1 — 90 minutes

# Mental mathematics

## Objectives

By the end of the session teachers will:

- have considered what is meant by mental mathematics;
- have considered some strategies for helping students to improve their skills in doing mental mathematics.

## Resources

### For the trainer

- Computer with data projector, Microsoft PowerPoint and Presentation 1.ppt
- Whiteboard and flipchart
- *Curriculum Standards for mathematics: Grades K to 12*

### For each teacher

- *Teacher's pack*
  Handouts 1.1–1.4
- *Curriculum Standards for mathematics: Grades K to 12*

## Session outline

| | | |
|---|---|---|
| **Pedagogical issues**<br>Slides 1.1–1.2<br>Handout 1.1 | Whole group presentation and discussion | 10 minutes |
| **Mental tests**<br>Slides 1.2–1.3<br>Handouts 1.2–1.3 | Whole group discussion and individual work | 30 minutes |
| **Explicit and implicit information 1**<br>Handout 1.4 | Whole group presentation and paired work | 30 minutes |
| **Explicit and implicit information 2**<br>Slides 1.4–1.8 | Paired work | 15 minutes |
| **Conclusion** | Summary | 5 minutes |

# Pedagogical issues

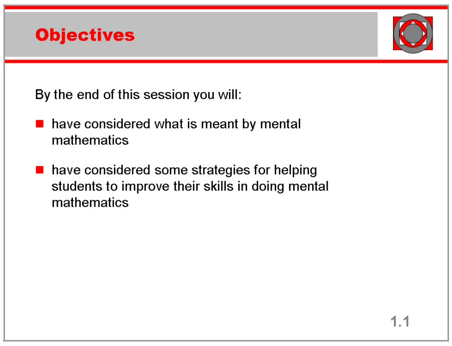
**10 minutes**

Before everyone arrives, brief any interpreter about the key points of the session.

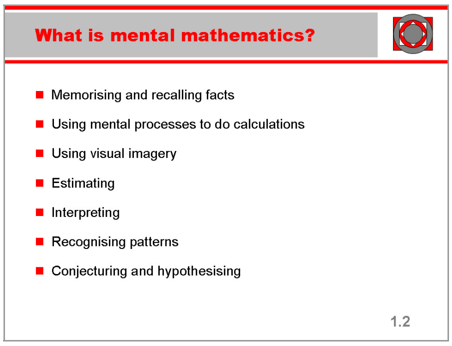
Put out copies of the **Teacher's pack**. Load **Presentation 1.ppt**.

Say that this session will focus on how to prepare students from Grade 7 through to Grade 12 to do mental mathematics.

Show the objectives for the session on **slide 1.1**.




Ask the question: 'What is mental mathematics?' Refer to **slide 1.2**.




Explain that everything we do in mathematics involves some form of mental work. Go through each bullet point in turn. Explain how all these things are done mentally.

- We memorise and recall factual knowledge. This includes number facts, formulae, properties of shapes, theorems, etc. This knowledge has become so internalised that normally we can recall it instantaneously.
- We develop mental processes and strategies for working mathematical problems out in our heads. The methods we use are often quite different from those we would use when we do written calculations. However, the methods are based on mathematical laws and conventions. We also use known facts to generate new facts.
- When teachers are planning their work they need to consider what visual images they are generating to aid learning. Consistent images that correctly represent the mathematics can be helpful. Some visual images can confuse and may even detract from students' learning. An image can be used as a way of generalising knowledge. Mathematics is a powerful subject because we can compact many facts and knowledge by generalising, using symbols and using categories. Good teachers are those who help students make links and connections to aid memorisation.
- We estimate measurements. We can do this because we have a sense of the standard measurements. Estimating is *not* guessing. We also use

approximate values to estimate an answer to a calculation when we only need a rough answer or when we want to check written or calculator answers.

- We interpret answers to questions with reference to the context. We inspect and interpret mathematical models such as graphs. We are able to synthesise the information using the knowledge we have acquired when dealing with similar problems. The more practice we have had the better skilled we become.
- We search for patterns in number and shape. Pattern recognition is an important mathematical skill.
- We hypothesise. We make conjectures. We explain and justify. We develop mathematical arguments.

Refer to **Handout 1.1**.

Explain that mental mathematics is mentioned explicitly only in Grades 7 and 8. Some minor references are made in other grades. However, mental mathematics is a central feature of the whole curriculum. In this session, we shall be looking at the broader picture.

There is a much greater emphasis on developing mental methods in number in Grades 1 to 6. This is necessary for students to be able to use these methods in Grades 7 to 12. It is essential to continue to practise recall of factual knowledge and use of mental processes. Teachers need to think about how they can use starter and plenary sessions to improve number skills and recall of factual knowledge.

## Mental tests

**30 minutes**

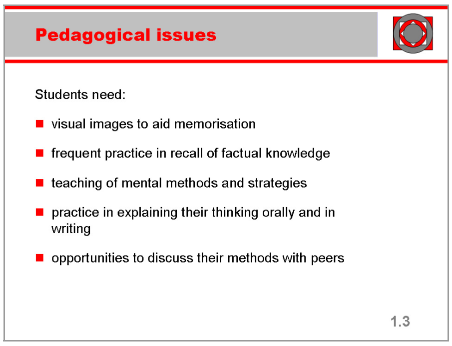
Refer teachers to **Handout 1.2**. Explain that you are going to ask them to do ten mental test questions that are similar to those used by students in England who are in the equivalent of Grade 8. Ask them to think about what they have to do in their heads when they answer the questions. Read the questions twice and give teachers sufficient time to answer. You are not testing their mathematical ability but giving them examples of what students are expected to do.

When teachers have completed the questions go through the answers quickly. Get teachers to look at each question on **Handout 1.3**. Ask them to refer again to **slide 1.2**, which remains displayed, and work together to identify which types of mental skill they were using.

Go through each question and take points from teachers. Note that some questions require a number of the skills to be used.

Refer to **slide 1.3**.




Demonstrate what is meant by each bullet point.

- Consider the 3 times table. Objects can be used to provide a visual image of each number fact. This aids memorisation. It also helps students to apply the result to a real-life situation. Students can only do inverse processes when the direct process has been fully mastered.
- Students need to speak mathematical facts aloud with peers and on their own. They need to write them down and test themselves and each other. They need to practise applying known facts to problems.
- Students need to be taught how to do calculations in their heads. Get teachers to do the multiplication $29 \times 41$ in their heads (answer: 1189). Ask them to reflect on their methods and write down what they did. Note the different ways of doing the calculation.

  Give another example such as $120 \div 19$ (Answer: 6 remainder 6). Elicit methods.

  Explain that mental methods require a secure knowledge of the laws of arithmetic. Students need these methods modelled for them. Say that the activity they have just done is a useful way of getting students to learn new methods from each other.
- Take the last two bullet points together. Teachers need to plan for opportunities for students to talk about mathematics. For example, they should have time to explain the way that they performed a calculation. They should explain and justify their answers. They should interpret graphs and diagrams. If students are given time to talk to each other before they present their answers and explanations to the class it will give them greater confidence.

## Explicit and implicit information 1

**30 minutes**

Refer to **Handout 1.4**. Explain that the multiplication tables provide explicit information that can be memorised. Help is given to students in the form of visual images such as those above. Help also comes in the form of chanting the tables out loud with the class followed by silent chanting. The 3 times table is called explicit information. The facts that we can derive from this table are called implicit information. One example of a new table derived from the given table is shown on the handout.

Now write $3 \times 4 = 12$ on the flipchart. This is *explicit* information. It is information stored in the memory by learning the 3 times table. When you know this fact you can access *implicit* information. Write examples on the flipchart such as:

300 × 4 = 1200, 3 × 40 = 120, 3 × 4000 = 12 000, 300 × 400 = 120 000

Ask teachers to give further examples.

Look again at 3 × 4 = 12. Point to the left-hand side. Ask: *How much is there there?* Point to the right hand side. Ask: *How much is there there?* Ascertain that there is twelve on each side of the equation. Say: *Look at 12.* [Point to both sides of the equation.] *How many 4s can you see?* (three) Demonstrate how the explicit information 3 × 4 = 12 provides the implicit information 12 ÷ 3 = 4 and 12 ÷ 4 = 3. It also provides further implicit information such as

120 ÷ 3 = 40, 12 000 ÷ 30 = 400, 12 ÷ 0.3 = 40, 1.2 ÷ 3 = 0.4

and:

120 ÷ 40 = 3, 1.2 ÷ 0.4 = 3, 1200 ÷ 40 = 30, 12 000 ÷ 4 = 3000, 0.12 ÷ 4 = 0.03

Ask teachers to give further examples.

Now look at the 53 times table. Given information like this is explicit information. Suggest that teachers could provide this table on the board at the start of a lesson. Ask teachers to write ten questions that can be answered using this table. Get teachers to give examples and write them on the flipchart.

Finally look at example C. This is a multiplication grid. It models the method of long multiplication. The grid provides explicit information. Ask teachers to write ten questions based on this grid.

Ask teachers to give their own examples of factual knowledge that can be derived from known facts. For example, if we know that 537 × 28 = 15 036, then we can derive 537 × 29. We know that we can do this because we know that numbers obey the distributive law of multiplication over addition:

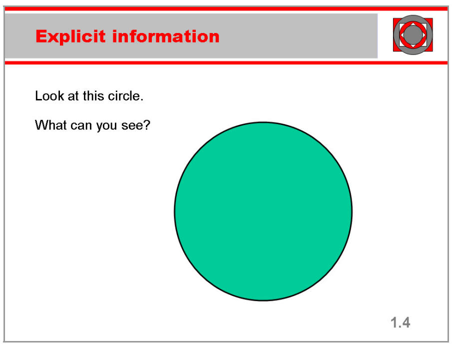
537 × 29 = 537 × (28 + 1) =537 × 28 + 537 × 1 = 15 036 + 537

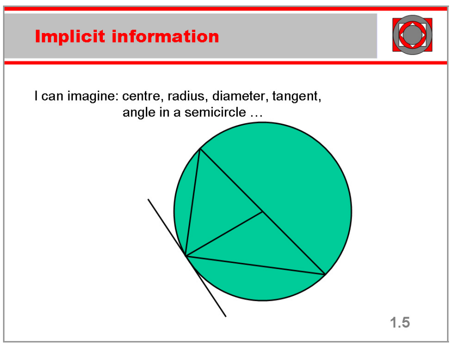
## Explicit and implicit information 2

**15 minutes**

Refer to **slides 1.4 to 1.8**. Go through each one.

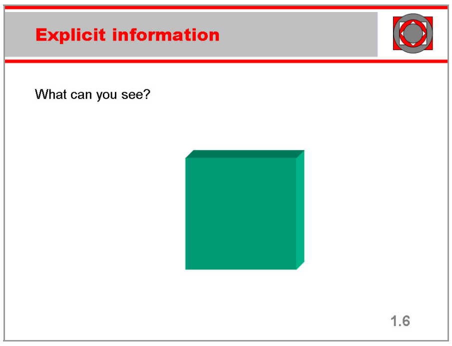
Ask teachers to look at the circle on **slide 1.4**. This is explicit information. What can teachers see in their heads? Get them to give answers. Then show **slide 1.5** and go through the implicit knowledge shown. Get more ideas, such as other circle theorems.

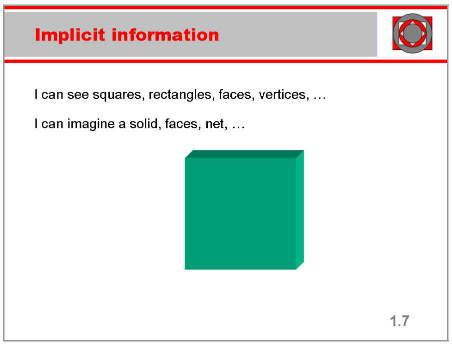


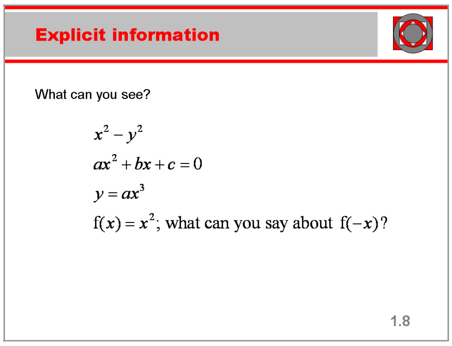

 

Ask teachers to look at the drawing on **slide 1.6**. Get them to explain what they can see. At first, insist that they only say what they can actually see as explicit information. Then get them to think about the possible things that the drawing could represent, such as a cube or a cuboid. Then look at **slide 1.7** and continue to ask questions such as: 'If I cut off a corner, what shape would you see?'




Ask teachers to look at the algebra on **slide 1.8**.




Remind teachers of the power of algebra in generalising rules etc. Ask them what they 'see' when they look at each one. Elicit numerical and graphical examples. Consider how knowing facts such as the difference of two squares enables us to calculate things such as 81 – 25 or find rules for Pythagorean triples. Get teachers to give further examples.

## Summary

**5 minutes**

Review the session by reflecting on:

- the meaning of mental mathematics;
- what is meant by implicit and explicit information;
- the need for careful planning of activities to improve students' skills;
- the need to practise mental arithmetic questions.