

Trigonometry

Objectives

By the end of this session teachers will:

- have considered some strategies for teaching trigonometry;
- have seen how doing practical work and using ICT can provide visual imagery that aids memorisation.

Resources

For the trainer

- Computer with data projector, Microsoft PowerPoint, *Autograph* and Presentation 7.ppt
- Whiteboard and flipchart
- Overhead projector (OHP)
- Graphics calculator with OHP connection
- A few spare protractors, rulers and compasses, and a pencil sharpener
- *Curriculum Standards for mathematics: Grades K to 12*
- Copies of the evaluation form for Day 2

For each group

- Graphics calculator

For each teacher

- *Teacher's pack*
Handouts 7.1–7.3, and evaluation form for Day 2
- Computer with *Autograph*
- Compasses, sharp pencil, protractor and ruler (to be brought by teachers)
- Sheet of plain paper and sheet of graph paper
- *Curriculum Standards for mathematics: Grades K to 12*

Session outline

Pedagogical issues Slides 7.1–7.2 Handout 7.1	Whole group presentation and discussion	10 minutes
The unit circle Handout 7.2	Practical workshop Task 1: Defining sine and cosine	35 minutes
Similar triangles	Whole group presentation and discussion	10 minutes
Problem solving Handout 7.3	Paired work on computers and/or graphics calculators Task 2: Solving problems	20 minutes
Conclusion Evaluation form for Day 2	Summary	5 minutes

Pedagogical issues

10 minutes

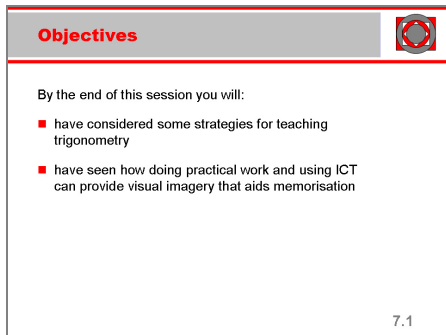
Say that this session will focus on:

- how to use practical tasks and graphical software to define sine and cosine;
- how to use these ideas in the solution of right-angled triangles.

Before everyone arrives, brief any interpreter about the key points of the session.

Load **Presentation 7.ppt**.

Show the objectives for the session on **slide 7.1**.



Slide 7.1: Objectives

By the end of this session you will:

- have considered some strategies for teaching trigonometry
- have seen how doing practical work and using ICT can provide visual imagery that aids memorisation

7.1

Refer to **Handout 7.1**.

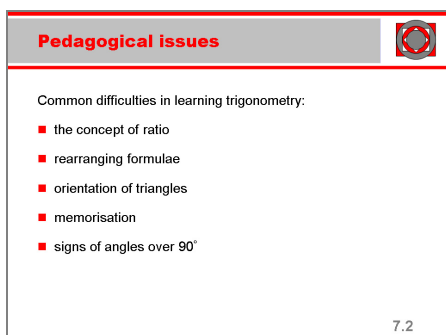
Say that teachers often assume that because students are first assessed in the application of trigonometry to right-angled triangles that this determines the order of teaching. However, students are better able to apply the ideas when they have a better understanding of the concepts.

This session suggests that using the unit circle to define sine and cosine helps students to learn how to apply sine and cosine to triangles. It also consolidates knowledge of similar triangles and proportionality.

Using graphical software can also help students' understanding but this should be used in conjunction with practical work. The speed of generating graphs with ICT can sometimes mask the underlying theory.

Show how most of the basic work on trigonometry is taught in Grade 9. But teachers need to prepare students in some of the necessary skills before this. The subject is developed in later years, particularly in Grade 11.

Refer to **slide 7.2**.



Slide 7.2: Pedagogical issues

Common difficulties in learning trigonometry:

- the concept of ratio
- rearranging formulae
- orientation of triangles
- memorisation
- signs of angles over 90°

7.2

Go through each point. Get teachers to add to your comments.

Background notes for trainers

- One of the most common ways of introducing sine, cosine and tangent of angles is with right-angled triangles and ratios. The problem with this is that there is no definition of the meaning of 'sine', etc. Students are expected to accept the facts they are given. They are presented with a set of ratios to memorise. Ratio and fractions are known to present difficulties.
- Practical methods provide visual imagery. Students are more likely to be able to see how to rearrange the formulae that they develop rather than attempting to rely on algebraic knowledge.
- Many texts present the right-angled triangle in one orientation. Students struggle when they see a triangle in a different position or as part of a composite shape. The practical work reinforces the need to look for the right angle and define the triangle.
- Learning trigonometry as ratios depends solely on memorisation. Being able to recall visual images helps memorisation.
- When students use rules to determine the sign associated with an angle they often forget or muddle the rules. The unit circle provides a visual image that can be recalled whenever a student needs to determine the sign.

The unit circle

35 minutes

Task 1: Defining sine and cosine

Refer everyone to **Handout 7.2**. Give out plain paper and graph paper. Tell teachers that they will need the ruler, compasses and protractor that they have brought with them.

Ask teachers to work in pairs and to follow the instructions on the handout. Tell them to consider the minimum number of measurements needed but not to tell their group.

Some teachers may have done this exercise before but it is important that they all do it and consider the teaching points. Get them to think about how they will plan for such an activity and when they might plan to do it. Teaching this in Grade 8, particularly with able students, would be good preparation for the trigonometric work expected in Grade 9. However, it could be left until Grade 9 if time is short. It will save time in the long term because students will have a better grasp of the subject.

Point out that students are revising other knowledge and skills while they do this exercise: measuring angles, measuring lines accurately to two decimal places, drawing graphs and looking for patterns.

When teachers have completed the measurements from the drawing ask them to check their results using their calculators. Then get them to draw the graphs on the graph paper. Ask them to think about the best way to draw the graphs and to consider what instructions they need to give their students. The scales on the axes will need discussion.

Recognising the symmetry of the circle and the repeated measurements is an important part of the teaching. Teachers need to have modelled the way of encouraging students to think when doing the task for themselves.

When everyone has completed the graphs, use *Autograph* to generate the graphs and get teachers to use computers or graphics calculators to do the same.

Teachers who complete the task early can do the extension tasks.

Similar triangles

10 minutes

Return to the unit circle. Show how to extract right-angled triangles with hypotenuse of length one unit. For any such triangle, the other two sides are given by the sine and cosine values of the angle at the centre of the circle.

Discuss how students need to do enough work with triangles of this type before moving on to triangles with different hypotenuse lengths. Teachers who think that their students like a challenge can get them to consider Pythagoras' theorem and deduce that $\cos^2 \theta + \sin^2 \theta = 1$.

Discuss:

- how to use similar triangles and scale factors to calculate lengths of sides of any right-angled triangle;
- how to develop the ratio from this;
- how students would use the ratios for future work (using the visual imagery to work out the ratios for themselves should they forget).

Relate the teaching of this work to Session 2 on proportional reasoning. Say that this reinforces the need for a common approach through the grades. Teachers of Grade 7 need to have an eye on what will be taught in Grade 9.

Solving problems

20 minutes

The timing of this session will depend on how quickly teachers have worked on the practical task. Refer them to **Handout 7.3**. This consists of a set of problems using trigonometry. Teachers should consider how to write good questions that will challenge students' thinking.

Summary

5 minutes

Review the session by considering:

- when and how to introduce trigonometry;
- the benefits of doing practical work;
- any issues concerning the planning and preparation of practical work;
- how visual imagery improves memorisation.

Finish the day by thanking everyone for their contributions. Invite them to complete the evaluation form for the second day and to give it to you before leaving.