

Understanding and using functions

Objectives

By the end of this session teachers will:

- have considered the significance of functions and their applications;
- have recognised and described key features of different functions;
- have generated some functions from situations and from other functions;
- have considered some of the pedagogical issues in teaching about functions.

Resources

For the trainer

- Computer with data projector, Microsoft PowerPoint, Autograph and Presentation 9c.ppt
- Whiteboard and flipchart
- Overhead projector (OHP) and blank acetate sheets
- Graphics calculator with OHP connection
- *Curriculum Standards for mathematics: Grades K to 12*

For each group

- Computer with *Autograph*
- Graphics calculator

For each teacher

- *Teacher's pack*
Handout 9c.1
- *Curriculum Standards for mathematics: Grades K to 12*

Session outline

Why functions? Slide 9c.1 Handout 9c.1	Whole group presentation and discussion Getting started	10 minutes
Describing functions Slide 9c.2 Handout 9c.1	Task 1: Describing functions Group work	25 minutes
Generating functions Slide 9c.3	Task 2: Generating functions Group work with computers or graphics calculators	25 minutes
Pedagogical issues Slides 9c.4–9c.5	Plenary	15 minutes
Summary	Summary	5 minutes

Why functions?

10 minutes

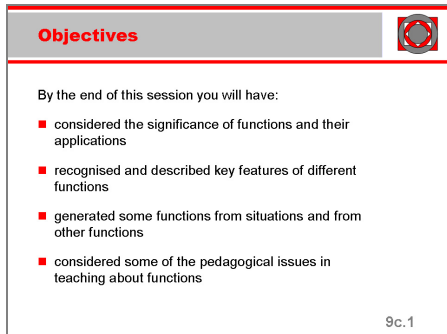
Explain that this session is about understanding and using functions, both within mathematics and in applications of mathematics. Show the objectives for the session on **slide 9c.1**.

Before the session starts, brief any interpreter about the key points of the session.

Make sure that each teacher has a copy of the standards.

Load **Presentation 9c.ppt**.

The main purpose of this session is to get teachers first to think generally about functions, their characteristics and their applications, and then to consider the pedagogical implications for effective teaching and learning about functions.



Objectives

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- considered the significance of functions and their applications
- recognised and described key features of different functions
- generated some functions from situations and from other functions
- considered some of the pedagogical issues in teaching about functions

9c.1

Say that the intention is for the group themselves to generate most of the ideas. Consequently specific mathematical examples will not be presented directly as part of the presentation.

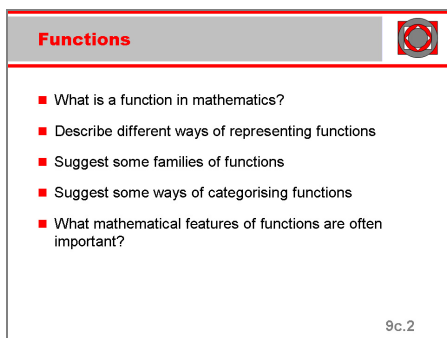
Refer everyone to **Handout 9c.1**. Ask teachers to work in pairs for a few minutes to suggest why they think functions are so central to mathematical thinking.

Bring the whole group together and record responses on a flipchart. Teachers should note the group's suggestions on the handout, both here and throughout this session.

Describing functions

25 minutes

Show **slide 9c.2**, and run through the bullets on the slide.



Functions

- What is a function in mathematics?
- Describe different ways of representing functions
- Suggest some families of functions
- Suggest some ways of categorising functions
- What mathematical features of functions are often important?

9c.2

Task 1: Describing functions

Ask teachers to work in school groups and to come up with suggestions for each of the bullets in the slide. Explain that you want teachers to be as precise as possible in their explanations and suggestions.

Allow 5 minutes for discussion, then share findings, summarising these on the flipchart. Look for at least some of the responses suggested below, and give the gist of several of them if necessary. They are *not* intended as the only suggestions or as the only formulations, but merely as possible responses.

Background notes for trainers

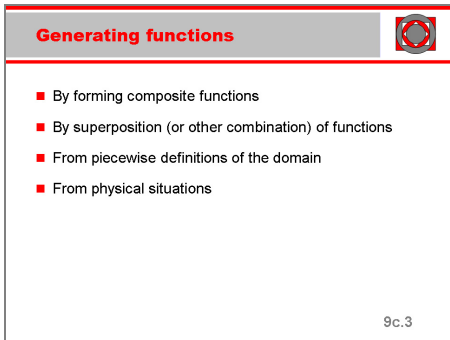
- What is a function in mathematics?
A function is a one-to-one or a many-to-one mapping, consisting of a set of elements called the domain and a rule that maps each of these elements to an element called its image. The set of all distinct images form the range of the function.
- Describe different ways of representing functions.
Algebraically, graphically or with a mapping diagram
- Suggest some families of functions.
Linear, polynomial (with further subdivisions such as quadratic, cubic, quartic, ...), rational, exponential, logarithmic, circular, etc.
- Suggest some ways of categorising functions.
Categorise by family of function type; by some mathematical properties, e.g. continuous everywhere, or discontinuous everywhere.
- What mathematical features of functions are often important?
Continuity, differentiability, function defined on the set of real numbers, function defined on the set of positive integers, asymptotic behaviour, intercept values in graphical representations, 'zeros' of the function, symmetry properties, gradient behaviour and turning points, stationary points, etc.

Generating functions

25 minutes

Task 2: Generating functions

Show **slide 9c.3**.



Generating functions

- By forming composite functions
- By superposition (or other combination) of functions
- From piecewise definitions of the domain
- From physical situations

9c.3

Ask teachers to work in school groups.

Tell the groups that they are to develop some specific examples for each bullet of the slide. Each group should develop at least one example under each heading, relating the examples to specific standards in particular grades. They should use computers/graphics calculators where relevant.

Allow them to ask any questions about their interpretation of the four bullets. If necessary, use an OHT to suggest an example or two.

Explain that each group should think about why they have chosen their particular examples, and be prepared to suggest direct extension activities.


Bring the groups together to share their thinking.


Pedagogy

15 minutes

Lead a whole group discussion on the pedagogic ideas that might be useful to teach about functions. Write suggestions on a flipchart.

Compare the suggestions with those on **slides 9c.4 and 9c.5**.

Pedagogical issues 1	
<ul style="list-style-type: none">■ Need for precision in describing functions and solving related equations■ Importance of the three-part nature of a function: the domain, rule and range■ Importance of using correct and relevant vocabulary■ Common features of members of families of functions■ Mathematical notations for functions■ Mathematical analysis of functions■ Alternative representations of functions	
9c.4	

Pedagogical issues 2	
<ul style="list-style-type: none">■ Making connections and comparisons■ Functions as mathematical models of physical contexts■ Further analysis and interpretation of functions in physical contexts■ Differentiation and extension activities■ The role of ICT in the teaching and learning of functions■ Assessment of understanding of the theory and applications of functions	
9c.5	

Summary

5 minutes

Ask teachers to state in turn what they have learned from this session.