



هيئة التعليم

EDUCATION INSTITUTE

Mathematics workshop 4

for teachers of Grades 1 to 6

Teacher's pack: Part 1

Developed for the Education Institute by CfBT

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Acknowledgements

The examples of division questions are drawn from or are adapted from the National Curriculum tests for England. The generosity of the Qualifications and Curriculum Authority for England for agreeing that these examples may be used is acknowledged gratefully.

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Introduction

Aims of the workshop

The purpose of this workshop is to continue to introduce the curriculum standards for mathematics and to discuss the implications for planning the curriculum, teaching, learning and assessment.

The workshops as a whole aim to help subject leaders and teachers to:

- become more familiar with the new curriculum standards;
- consider the implications of the standards for planning, teaching and assessment;
- start or refine the planning of a mathematics scheme of work based on the standards, and related lesson plans;
- support colleagues as they implement the standards.

After this workshop, you will need to:

- discuss with your school principal, and SSO support team, any implications of the workshop;
- feed back to other colleagues what has been learned on the workshop and discuss any action needed;
- put into practice as much as possible of what you have learned.

Workshop programme

Day 1: Problem solving and data handling

08:00	Registration	
Session 1 08:30–10:00	Problem solving 1	90 minutes
Session 2 10:30–12:00	Problem solving 2	90 minutes
Session 3 13:00–14:30	Data handling 1	90 minutes
Session 4 15:00–16:30	Data handling 2	90 minutes

Day 2: Division

10:45	Registration	
Session 5 11:10–12:30	Division 1	80 minutes
Session 6 13:30–14:50	Division 2	80 minutes
Session 7 15:10–16:30	Planning lessons on division	80 minutes

Day 3: Early years, geometry and measures

10:45	Registration	
Session 8 11:10–12:30	Early years	80 minutes
Session 9 13:30–14:50	Geometry and measures 1	80 minutes
Session 10 15:10–16:30	Geometry and measures 2	80 minutes

Day 4: Aspects of number

10:45	Registration	
Session 11 11:10–12:30	Division and assessment	80 minutes
Session 12 13:30–14:50	Direct proportion	80 minutes
Session 13 15:10–16:30	Numeracy and ICT	80 minutes

Day 5: Using ICT

10:45	Registration	
Session 14 11:10–12:30	Using a calculator	80 minutes
Session 15 13:30–14:50	The interactive whiteboard	80 minutes
Session 16 15:10–16:20	Summing up	70 minutes

Objectives of each session

Day 1: Problem solving and data handling

Sessions 1 and 2: Problem solving 1 and 2

By the end of the sessions teachers will:

- know some strategies that students could use to solve mathematical problems;
- have considered what ‘being systematic’ means;
- know some approaches to teaching problem solving;
- have discussed how students might use reasoning to justify their conjectures and conclusions;
- know how a teacher can guide students’ reasoning;
- know how students might record their reasoning.

Session 3 and 4: Data handling 1 and 2

By the end of the sessions teachers will:

- be familiar with the standards for data handling from Grade 1 to Grade 6;
- understand the key elements of data handling;

- be familiar with a range of problem-solving activities involving collecting, organising, representing and interpreting data, and drawing conclusions;
- have discussed ways of using ICT in data handling;
- have considered strategies for assessing students' knowledge and understanding of data handling.

Day 2: Division

Sessions 5, 6 and 7

By the end of the sessions teachers will:

- have reviewed progression in division in Grades 2 to 6;
- know how to use models, images and language in the teaching of division;
- know how a number line can be used to teach division, including representing the quotient as a fraction;
- have considered approaches to mental and written division calculations;
- know how students can be helped to learn division facts;
- have discussed approaches to the teaching of short division;
- have considered some lessons and resources to support the teaching of division.

Day 3: Early years, geometry and measures

Session 8: Early years

By the end of the session teachers will:

- understand the Kindergarten standards for mathematics;
- be aware of some features of effective teaching and learning in Kindergarten and Grade 1.

Sessions 9 and 10: Geometry and measures 1 and 2

By the end of the sessions teachers will:

- be familiar with the standards for geometry from Grade 1 to Grade 6;
- understand the key elements of transformations;
- understand the progression in work on area;
- be familiar with a range of problem-solving activities to use with students to support their understanding of shape, space, constructions, perimeter, area and volume.

Day 4: Aspects of number

Session 11: Division and assessment

By the end of the session teachers will:

- have analysed test questions on division as an aid to assessment;
- have considered the errors that students may make with division and the implications for teaching.

Session 12: Direct proportion

By the end of the session teachers will:

- understand direct proportion and know how to teach it.

Session 13: Numeracy and ICT

By the end of the session teachers will:

- be familiar with some small programs that support development of number skills in the primary grades.

Day 5: Using ICT

Session 14: Using a calculator

By the end of the session teachers will:

- have considered how a calculator can support students' understanding in mathematics;
- be familiar with the use of a calculator in the mathematics standards;
- be aware of a range of problem-solving activities involving the use of a calculator.

Session 15: The interactive whiteboard

By the end of the session teachers will:

- have viewed and discussed some ICT resources;
- have discussed the benefits of interactive teaching programs and specialist software.

Session 16: Summing up

By the end of the session teachers will:

- have watched videos tailored to the local context;
- have reflected on the workshop.

Materials you need to bring to the workshop

When you come to the workshop, please bring with you:

- *Curriculum Standards for mathematics: Grades K to 12*;
- a writing pad on which to make notes;
- a ruler.

Each day you will need to bring with you the *Teacher's pack: Part 2* that you will be given on the first day of the workshop.

Tasks to do before the workshop

Before coming to the workshop, you are asked to complete the tasks given to you at the end of the previous workshop. These tasks were:

- Read the article *Beginning division*, which is reproduced again on the next few pages of this booklet.
- Pick a set of questions on division from those in *Division questions*, which follow the article *Beginning division*. Choose questions suited to the students in your class. The questions may be drawn from those for a higher grade or a lower grade if you wish. If necessary, translate the questions into Arabic.

You may want to give one group of students a set of easier questions and another group a set of harder questions. If you wish, you can give a set of questions to every student in your class, or you may prefer to give a set of questions to just some of the students. Try to give the questions to at least 12 students.

Ask the students to do the questions at a convenient time, doing the work on pieces of plain paper.

Collect in the work and mark it. Identify three different examples of errors that students have made. Paste each example on to a separate sheet of plain A4 paper, making sure that it is anonymous (i.e. don't identify your school or the student). **Bring the three sheets of paper with you to the workshop.**

The questions are not meant to be a test. They are intended to help you to identify examples of the kinds of errors that students make when they tackle division questions. There is no time limit for the questions.

You may want to follow up the work that the students have done, either immediately or in due course. For example, you could ask students to correct any careless mistakes that they have made, or do some specific teaching to put right evident misunderstandings. You could also use the information you have gathered to adjust your plans for the next teaching unit on calculations.

Beginning division

Some vocabulary associated with multiplication and division

- **Product**

A product is the result of multiplying numbers together. For example, the product of 2, 3 and 4 is 24, or $2 \times 3 \times 4$; the product of 121 and 7 is 847, or 121×7 .

- **Quotient**

A quotient is the result of dividing one number by another. For example, if 45 is divided by 9, the quotient is 5.

- **Dividend**

A number that is to be divided by another is called the dividend. For example, in $246 \div 3$, the dividend is 246.

- **Divisor**

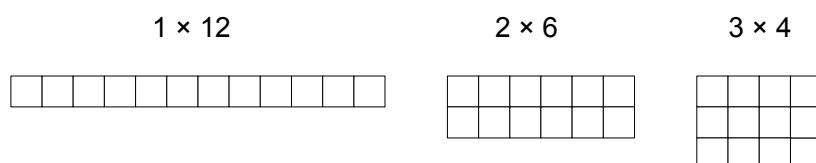
The number by which the dividend is to be divided is called the divisor. For example, in $246 \div 3$, the divisor is 3.

- **Multiple**

The number a is a multiple of the number b if b divides a exactly. For example, 5, 10, 15, 20, 25, ... are all multiples of 5.

- **Factor**

The factors of a number are all the numbers that divide into it exactly. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. The factors of 17 are 1 and 17. Factors always occur in pairs. A useful model for thinking about factors is a rectangular array. For example, the pairs of factors of 12 are:



A **prime number** is a number that has exactly two factors, itself and 1.

- **Inverse**

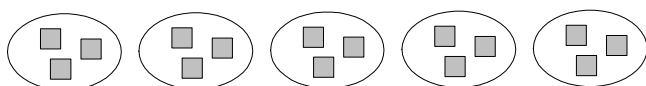
Multiplication and division are inverse processes. This means that since $5 \times 8 = 40$, then $40 \div 8 = 5$. Dividing by 8 ‘undoes’ the effect of multiplying by 8.

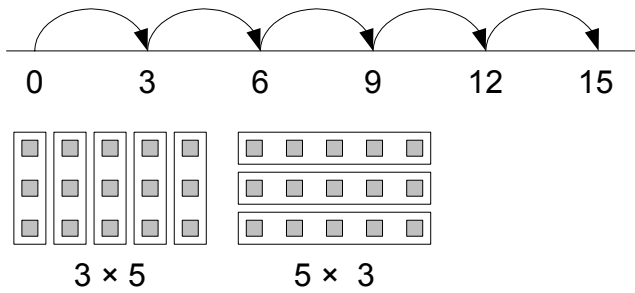
Images or models of multiplication

Two kinds of multiplication models are:

- repeated addition;
- scaling.

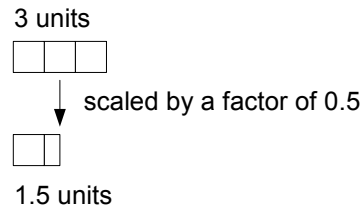
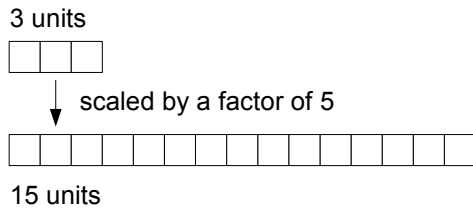
Repeated addition is the idea that multiplication means ‘so many groups of’. If I have ‘5 lots of 3 bricks’, then the answer to ‘How many bricks are there altogether?’ is 3×5 , or $3 + 3 + 3 + 3 + 3$. We read 3×5 as ‘3 multiplied by 5’, since the 5 is operating on the 3.





The last of these three models, in which a column of 3 squares is repeated 5 times, is useful for showing that $3 \times 5 = 5 \times 3$.

Scaling involves increasing a quantity by a scale factor. Multiplication by 5 entails scaling a quantity by a factor of 5, and making it 5 times as big. Multiplication by a number less than 1 corresponds to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to the multiplication 3×0.5 .

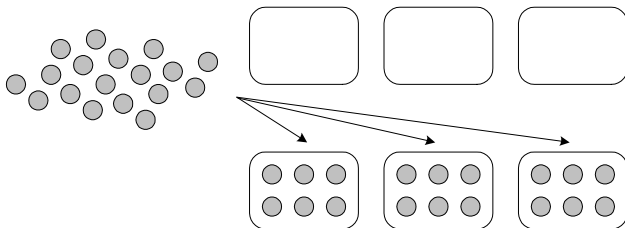


Images or models of division

The most common kinds of division models are:

- equal sharing;
- grouping.

Equal sharing occurs when a quantity is shared out equally into a given number of portions, and we work out how many there are in each portion. For example, if 18 marbles are shared out equally among 3 students, the calculation $18 \div 3$ can be modelled as:

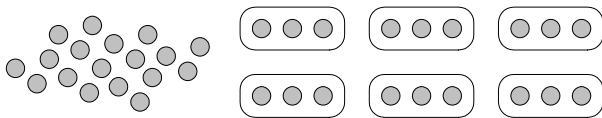


This model helps to answer questions such as: ‘Divide 18 into 3 equal groups. How many are there in each group?’ The associated calculation is $18 \div 3$ and the answer is 6, as there are 6 in each group. We read $18 \div 3$ as ‘18 divided between 3’. The 3 is operating on the 18.

Equal sharing corresponds to finding a fraction. In the example of the 18 marbles, each of the three equal groups is one third of the marbles, so $\frac{1}{3}$ of 18 is equivalent to $18 \div 3$.

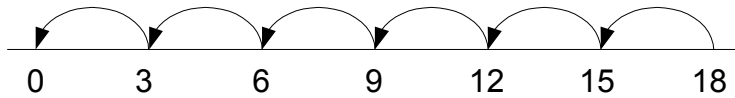
Grouping occurs when we are asked to find how many groups of a given size are equivalent to the original quantity. For example, if we determine the number of

groups of 3 marbles in a set of 18 marbles, the calculation $18 \div 3$ can be modelled as:

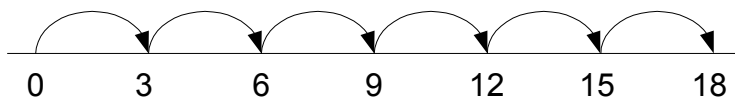


This model helps to answer questions such as: ‘Divide 18 into groups of 3. How many groups are there?’

The grouping model of division may involve *repeated subtraction* (‘How many groups of 3 can I get from a group of 18?’):



or *repeated addition* (‘How many groups of 3 do I need to get a group of 18?’).

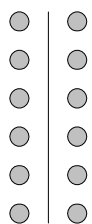


Either of these can be summarised by: ‘How many 3s make 18?’

The relationship between grouping, sharing and finding fractions

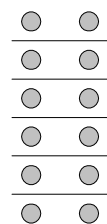
The models of division can be compared like this.

$12 \div 2$ (sharing or halving)



How many in one column?

$12 \div 2$ (grouping)



How many rows are there?

The questions ‘How many in one column?’ and ‘How many rows are there?’ are effectively the same question and give the same answer, since they are underpinned by the commutative law of multiplication: $6 \times 2 = 2 \times 6$. So when presented with a calculation we can choose to represent it in the way that is most appropriate and beneficial.

Advantages and disadvantages of different models of division

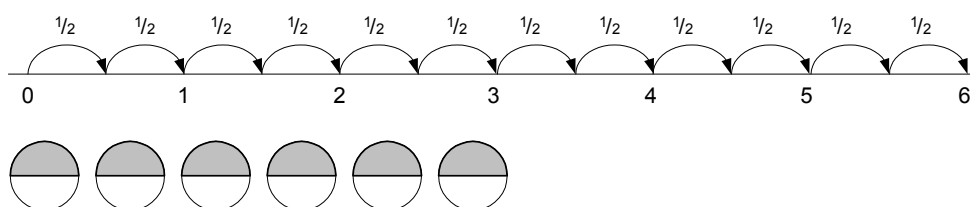
In Grades 1 and 2 students are generally introduced to division through practical activities that initially involve sharing, and later grouping, of objects.

Young students often engage in sharing activities of their own; sharing of equipment is a common feature of many lessons. Equipment is sometimes shared ‘1 between 2’ so that the divisor is larger than the dividend. Some sharing results in a remainder, for example, when things like crayons or sweets do not go round exactly. These activities lay the foundations for later work.

Sharing can give students an early understanding of fractions. For example, sharing between 2 using a ‘one for you, and one for you’ approach promotes the idea of halving. Students can picture the halving process and see the one-to-one correspondence between the objects in the two halves. Later, having shared 20 between 2 and visualised the two 10s, students can share out 24 by splitting it into $20 + 4$, and sharing out the extra 4.

But equal sharing as a model for division has limitations. For example, sharing 1200 among 200 is impossible to visualise and even sharing 32 among 7 is difficult. Recording using 32 tally marks is inefficient and leads to mistakes. Recognising what to do with a remainder presents problems when division is based on sharing.

Grouping is more efficient, if students have a good grasp of addition and subtraction. It also establishes a firmer basis upon which to build understanding of division. For example, thinking of $6 \div \frac{1}{2}$ as ‘share 6 between $\frac{1}{2}$ ’ is meaningless. But students who are accustomed to thinking of $6 \div 2$ as grouping to find out ‘How many 2s make 6?’ will have little difficulty in thinking of, and modelling, $6 \div \frac{1}{2}$ as ‘How many halves make 6?’, and interpreting $6 \div \frac{1}{2}$ as 6 divided into halves.



The language of division

It is important that students read a calculation such as $18 \div 3$ in a way that includes the word ‘divide’. In a grouping context, the phrasing will be ‘18 divided by 3’. In a sharing context, the phrasing will be ‘18 divided between 3’. In a context of finding a fraction (one third), the phrasing will be ‘18 divided into 3 (equal groups)’. The wording links to the model of division and helps students to visualise what is happening.

Contexts in which students meet division

Measurement contexts provide examples of **equal sharing** (though the situations are rather contrived). The number of equal shares is known and we want to know how much there is in each share: for example, when we share 750 g of food equally between 5 bowls, or cut 750 mm of ribbon into 5 equal lengths, or pour 750 ml water into 5 glasses so that each glass has the same amount.

The context of money offers examples that are more realistic: for example, sharing a bill for food in a restaurant, or the cost of a holiday. There are also examples where items are sold in multiple packs and we want to know the cost per item (*per* means ‘for each’). If 4 bars of soap are sold for QR 14, the cost per bar is found by the division $14 \div 4$. Similarly, to find the cost per DVD of 5 DVDs at QR 180, or the average speed of a car that travels 180 kilometres in 5 hours, we calculate $180 \div 5$.

Examples of **grouping** occur when a school wants to know how many classes will result if 180 students are organised into classes of 30, or when someone putting

chairs in a concert hall wants to know how many rows will be formed if 70 chairs are placed in rows of 14.

Money and measurement contexts offer examples of grouping, such as: ‘How many sweets at QR 1.50 can I buy with QR 7.50?’ or: ‘How many 150 ml servings of fruit juice can I get from this 750 ml carton?’ Each of these can be represented by $750 \div 150$ and modelled with repeated subtraction. Questions such as: ‘How many lengths of 50 cm can I cut from 10 metres of ribbon?’ or: ‘If I save QR 50 per month, how many months will it take me to save QR 1000?’ can be represented by $1000 \div 50$ and modelled with repeated addition.

Teaching students to recall division facts

Over Grades 1 to 5, students should build up their knowledge of multiplication facts and learn them ‘by heart’. They also need to know by heart the corresponding division facts. The way that multiplication tables are chanted can be significant in helping to learn division facts. If a table is chanted as:

One times six is six.
Two times six is twelve.
Three times six is eighteen.
Four times six is twenty-four ...

it is not particularly helpful when it comes to thinking about $24 \div 6$, and the question ‘How many 6s make 24?’ However, if the table is chanted as:

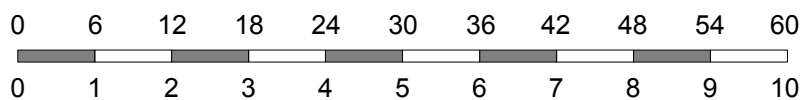
One six is six.
Two sixes are twelve.
Three sixes are eighteen.
Four sixes are twenty-four ...

the answer to the question ‘How many 6s make 24?’ relates more closely to the wording of the chant.

Occasional chanting of division tables can help to establish both the knowledge of division facts in their own right and the use of the phrase ‘divided by’. For example:

Six divided by six is one.
Twelve divided by six is two.
Eighteen divided by six is three ...

Chanting of division tables can be related to the well-established activity of counting and supported with a counting stick. This helps to establish the relationship between the increasing steps and the corresponding quotients.



Using a vertical counting stick makes a more direct correspondence to a recorded division table.

0	0	$0 \div 6 = 0$
6	1	$6 \div 6 = 1$
12	2	$12 \div 6 = 2$
18	3	$18 \div 6 = 3$
24	4	$24 \div 6 = 4$
30	5	$30 \div 6 = 5$
36	6	$36 \div 6 = 6$
42	7	$42 \div 6 = 7$
48	8	$48 \div 6 = 8$
54	9	$54 \div 6 = 9$
60	10	$60 \div 6 = 10$

Another way to help students to learn division facts is to ask them to identify which facts in a table are easy to remember, and why, and which are hard to remember, and then to repeat the 'harder' facts three times each.

Asking a selection of mixed questions related to the table after it has been chanted helps to establish the relevant vocabulary.

- What is 8 multiplied by 3?
- What is 27 divided by 3?
- What is the product of 7 and 3?
- What is one third of 15?
- What is 3 squared?
- How many threes make 21?
- What is the next multiple of 3 after 30? How did you work it out?
- I multiply a number by 3. The answer is 30. What is the number? How did you work it out?
- What is the remainder when 29 is divided by 3? How did you work it out?
- I divide a number by 3. The answer is 6 remainder 1. What is the number? How did you work it out?

Summary

- Students in Grades 1 to 6 will normally meet multiplication as repeated addition. They will begin to experience examples of scaling in Grades 5 and 6.
- Rectangular arrays are helpful for modelling the commutative law of multiplication.
- It is important to establish the commutative law for multiplication firmly in students' minds and to encourage them constantly to use it. This makes it much easier for students to learn and recall the facts in multiplication tables.
- Students should learn division facts 'by heart'.

- Equal sharing is one model of division. This model helps to answer questions such as: ‘Divide 18 into 3 equal groups. How many are there in each group?’ It can be represented by giving out objects on a ‘one for you, one for you’ basis. However, sharing has limitations when it comes to later work on division.
- Finding a fraction like one half, one third, one quarter, ... is equivalent to equal sharing.
- The division model of grouping occurs as repeated subtraction or repeated addition (counting up to reach a target). It helps to answer questions such as: ‘Divide 18 into groups of 3. How many groups are there?’ This model of division has the advantage that it can be represented on the familiar number line.
- Teachers and students should read the calculation $18 \div 2$ as ‘18 divided by 2’, or ‘18 divided between 2’ or ‘18 divided into 2’, depending on the context.
- Students should experience the different types of division in a wide range of practical, relevant contexts.

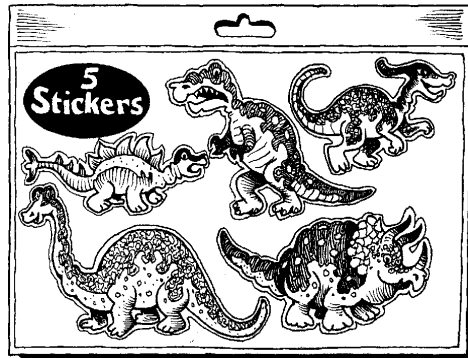
Self-assessments

- 300 eggs are packed in boxes of 12. How many boxes are used?
In the division calculation that represents this problem, which number is the dividend, which is the divisor and which is the quotient?
- If you save QR 13.50 per week, for how many weeks would you need to save in order to buy a tennis racquet costing QR 378?
What calculation would you enter on a calculator?
What model of division does this question represent?
- Make up a problem that corresponds to the division $30 \div 5$, based on equal sharing in the context of shopping.
- Make up a problem that corresponds to the division $30 \div 5$, based on grouping (repeated subtraction) in the context of shopping.

Division questions (Grade 2)

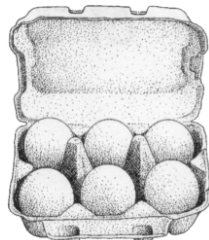
1 20 students sit at tables in groups of 4.
How many groups will there be?

2 A packet of stickers costs 50 dirhams.



How many packets can you buy for 3 riyals?

3 There are 20 eggs.
A box holds 6 eggs.



How many boxes are needed to hold all the eggs?

Division questions (Grade 3)

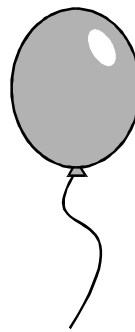
1 Circle each number that has a remainder of 2 when divided by 5.

27 15 26 45 32 24

2 Write in the missing number.

$$2 \times \boxed{} = 18$$

3 There are 5 balloons in a packet.



Roza needs 45 balloons.

How many packets does she need?

4 Omar has 16 cards.

He gives a quarter of his cards to Mosa.

How many cards does Omar give to Mosa?

5 There are 24 drinking straws in a box.
The straws are red, green and blue.

There is the same number of each colour.
How many red straws are in the box?

Division questions (Grade 4)

1 $800 \div 40 =$

2 Write in the missing number.

$\div 4 = 21$

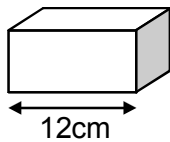
3 230 students need to travel by coach.
Each coach holds 50 students.

How many coaches are needed?

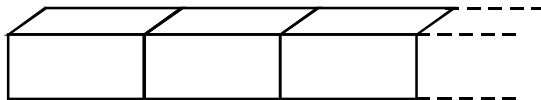
4 One length of a swimming pool is 25 metres.

How many lengths are there
in a 150 metre race?

5 Said has some bricks.
Each brick is 12 cm long.



Said makes a line of bricks 132 cm long.



How many bricks does he use?

Division questions (Grade 5)

No calculator

1 Calculate:

$$847 \div 7 = \boxed{}$$

2 Eggs are put in trays of 12.
The trays are packed in boxes.
Each box contains 180 eggs.

How many trays are in each box?

3 Some boys go camping.
There are 70 boys.
Each tent takes up to 6 boys.

What is the least number of tents they will need?

Calculator allowed

4 Write in the missing number.

$$32.45 \times \boxed{} = 253.11$$

5 Tickets for an exhibition cost QR 4.50 each.
Selling tickets raises QR 229.50.

How many tickets are sold?

Division questions (Grade 6)

No calculator

1 Calculate:

$$924 \div 22 = \boxed{}$$

2 Write in the missing digit.

$$\boxed{}92 \div 14 = 28$$

3 Sharifa buys 24 figs for QR 6.

What is the cost of each fig?

Calculator allowed

4 Write in the missing digit. The answer does not have a remainder.

$$\begin{array}{r} 26 \\ 3 \overline{) \boxed{}8} \end{array}$$

5 There is 60 g of rice in one portion.

How many portions are there in a 3 kg bag of rice?

6 427 students visit the Oryx Farm.

They go in groups of 15. One group has less than 15.

Every group of students has one adult with them.

How many adults will need to go?

