هيئة التعليم
EDUCATION INSTITUTE

Mathematics workshop 4

for teachers of Grades 1 to 6

Teacher’s pack: Part 3
Handouts for Session 7

Developed for the Education Institute by CfBT

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Continuing division

Prerequisite skills for mental multiplication and division

Before they develop further multiplication and division skills, students need to be able to do the following.

- Add and subtract accurately and efficiently.
- Recall quickly all the facts in the multiplication tables to 10 × 10 and the division tables to 100 ÷ 10.
- Understand the effect of multiplying and dividing whole numbers by 10 and 100 (and later 1000).
- Recognise factors of numbers (e.g. that $15 = 5 \times 3$, or that $40 = 10 \times 4$).
- Derive other results from multiplication and division facts and multiplication or division by 10 or 100. For example, from $4 \times 8 = 32$, deduce that:

$$\begin{array}{rcrcr@{\qquad\qquad}rcrcr}
4 & \times & 80 & = & 320 & 4 & = & 320 & \div & 80 \\
4 & \times & 800 & = & 3200 & 4 & = & 3200 & \div & 800 \\
40 & \times & 8 & = & 320 & 40 & = & 320 & \div & 8 \\
40 & \times & 80 & = & 3200 & 40 & = & 3200 & \div & 80 \\
40 & \times & 800 & = & 32\,000 & 40 & = & 32\,000 & \div & 800 \\
400 & \times & 8 & = & 3200 & 400 & = & 3200 & \div & 8 \\
400 & \times & 80 & = & 32\,000 & 400 & = & 32\,000 & \div & 80
\end{array}$$

  and so on.
- Understand how the principles of the commutative, associative and distributive laws apply (or do not apply) to multiplication and division.

Notice how the inverse operation is being applied: if $a \times b = c$, then $a = c \div b$.

It also helps to notice that the total number of zeros in the two numbers being multiplied is the same as the number of zeros after the 32 in the product.

The laws of arithmetic

Primary students don't need to know the names of these laws, or to see them expressed algebraically, but they do need to understand how the laws work in practice if they are to become proficient at multiplication and division.

Commutative law of multiplication: $a \times b = b \times a$
Associative law of multiplication: $(a \times b) \times c = a \times (b \times c)$
Distributive law of multiplication: $(a + b) \times c = (a \times c) + (b \times c)$
$(a - b) \times c = (a \times c) - (b \times c)$

Written formally, the laws of arithmetic can look awesome. But anyone who does a multiplication calculation will probably use the laws unconsciously.

For example, take the calculation 5 × 18. Most people prefer to think of this as 18 fives, rather than 5 eighteens, because the 5 times table is easier than the 18 times table. The **commutative law** of multiplication allows the order of the numbers to be switched as much as you wish.

To do 18 × 5, you could think of 18 as 9 × 2, so the calculation becomes 9 × 2 × 5. You could choose to do 2 × 5 first, to get 10, then work out 9 × 10 = 90. This method uses the **associative law** of multiplication – the 2 is associated with the 5, rather than the 9, to make the calculation easier.

But you may prefer to calculate 18 × 5 by splitting 18 into 10 + 8, and multiplying the 10 and 8 separately by 5. This gives you 50 and 40, which add up to 90. This method uses the **distributive law** of multiplication. The multiplication by 5 is distributed across the addition of 10 and 8.

Similarly, if 18×5 is thought of as $(20 - 2) \times 5$, the multiplication by 5 can be distributed across the subtraction $20 - 2$, giving $(20 \times 5) - (2 \times 5)$, or $100 - 10 = 90$.

We also want students to realise that division:

- is not commutative: for example, $12 \div 3$ is not equal to $3 \div 12$;
- is not associative: for example, $(24 \div 6) \div 2$ is not equal to $24 \div (6 \div 2)$, since $(24 \div 6) \div 2 = 4 \div 2 = 2$, and $24 \div (6 \div 2) = 24 \div 3 = 8$;

but that it can be distributed across addition and subtraction.

Distributive law of division:

$$(a + b) \div c = (a \div c) + (b \div c)$$
$$(a - b) \div c = (a \div c) - (b \div c)$$

So provided that c is not zero, since division by zero is not possible, if a dividend is the sum of two numbers $(a + b)$, you can divide each of a and b separately by the divisor c and add the results. For example:

$$96 \div 3 = (90 + 6) \div 3 = (90 \div 3) + (6 \div 3) = 30 + 2 = 32$$

Similarly, if a dividend is the difference of two numbers $(a - b)$, you can divide each of a and b separately by the divisor c and find the difference between the results. For example:

$$297 \div 3 = (300 - 3) \div 3 = (300 \div 3) - (3 \div 3) = 100 - 1 = 99$$

Some strategies for mental multiplication and division

Most mental methods of multiplication and division involve either **factorising** or **partitioning**. The calculation may be done entirely mentally, or informal recording may be used to support thinking. Recording is also used to explain to someone else how the calculation has been done.

Factorising is particularly useful for multiplying or dividing by multiples of 10. For example, to calculate 6×40, factorise 40, then calculate $(6 \times 4) \times 10$. To divide by 40, divide first by 10, then by 4.

Other useful mental methods to know are for multiplying by 50 (multiply by 100, then divide by 2) and for multiplying by 25 (multiply by 100, then divide by 4). Dividing a number by 4, 8 or 16 is merely a matter of continued halving of the number as many times as is necessary.

The mental calculation of a product such as 26×15 can be tackled by factorising 26 as 13×2 and applying the associative law.

$$\begin{aligned} 26 \times 15 &= (13 \times 2) \times 15 \\ &= 13 \times (2 \times 15) \\ &= 13 \times 30 \\ &= 390 \end{aligned}$$

A calculation like 43×6 might be tackled by **partitioning** 43 into $40 + 3$. Informal recording of the use of the distributive law might be, for example:

$$\begin{tikzcd} & 43 & & \\ 40 \arrow[dd] & + & 3 \arrow[dd] & \\ & & & \times 6 \\ 240 & + & 18 & = 258 \end{tikzcd}$$

×	40	3
6		

×	40	3
6	240	18

240 + 18 = 258

$$\begin{aligned} 43 \times 6 &= (40 + 3) \times 6 \\ &= (40 \times 6) + (3 \times 6) \\ &= 240 + 18 \\ &= 258 \end{aligned}$$

Partitioning is also useful in a division such as 84 ÷ 7, but here a refinement to one of the steps is needed. This time, the dividend of 84 is split not into 80 + 4 but into the greatest multiple of 7 tens or 70, plus the rest. So 84 is split into 70 + 14, then each part is divided by 7.

$$\begin{tikzcd} & 84 & & \\ 70 \arrow[dd] & + & 14 \arrow[dd] & \\ & & & \div 7 \\ 10 & + & 2 & = 12 \end{tikzcd}$$

×		
7	70	14

×	10	2
7	70	14

10 + 2 = 12

$$\begin{aligned} 84 \div 7 &= (70 + 14) \div 7 \\ &= (70 \div 7) + (14 \div 7) \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

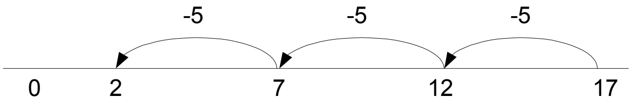
Sometimes it can be useful to round up the dividend to a multiple of 10 that is a multiple of the divisor and then to use the distributive law for division over subtraction. For example:

$$\begin{aligned} 116 \div 4 &= (120 - 4) \div 4 \\ &= (120 \div 4) - (4 \div 4) \\ &= 30 - 1 \\ &= 29 \end{aligned}$$

Many students can apply the distributive law for multiplication with confidence. But this is not the case for the distributive law for division over addition and subtraction. One reason for this may be that the distributive law for division may go unrecognised by teachers. Another may be that the teaching and learning of mental methods of division, stressing the correspondence to mental methods of multiplication, are not given enough attention to ensure that students' application of the distributive law for division becomes second nature.

Remainders

The idea of stepping back to zero reflects the idea of repeated subtraction that underpins the grouping model of division. It is also a helpful way to teach Grades 2 and 3 how to find remainders after division. For example, 17 ÷ 5 can be represented by:



We cannot step back to zero. From 17 we can take 3 steps of 5 to 2. The 2 is the remainder after this division.

The approach of steps back on a number line can be applied to teach Grade 4 how the remainder can be represented in a fraction. The remaining 2 is part of the step of size 5, represented as ⅖.The answer to 17 ÷ 5 is 3⅖, or 3 full steps of 5 and two fifths of a step. By Grade 4, most students should be able to use their knowledge of the equivalence between fractions and decimals to give the answer to 17 ÷ 5 as either 3⅖ or 3.4.

Examples of the kinds of division calculations which students in Grades 4, 5 and 6 should tackle mentally, if necessary with informal recording to support their thinking, are:

no exchange, no remainder	69 ÷ 3	808 ÷ 4
no exchange, with remainder	68 ÷ 3	
with exchange, no remainder	98 ÷ 7	128 ÷ 4
with exchange, with remainder	47 ÷ 3	
decimal dividend	7.5 ÷ 5	0.12 ÷ 3

Interpreting remainders in context

The interpretation of a remainder may depend on the context. Take the division 60 ÷ 8, with the answer 7 remainder 4, 7½ or 7.5.

Consider a grouping context. If 60 pens are packed into boxes holding 8 pens, the remainder of 4 represents the surplus pens after 7 boxes of 8 pens have been packed. The fraction ½ and the 0.5 in 7.5 represent the fraction of a box that would be taken up by the 4 remaining pens. So the remainder represents the left-over pens and the fraction and the figure after the decimal point represent 'a bit of a box'. The answer might be expressed as 7 full boxes with 4 pens left over, or 7.5 or 7½ full boxes.

Now consider an equal sharing context. If 60 pens are shared equally among 8 people, each person gets 7 pens. The remainder of 4 represents the 4 pens left over after the sharing has been done. The fraction ½ and the 0.5 in 7.5 represent the proportion of a pen that each person would get if it were possible to share out the remaining pens equally.

Whether or not you can share out a remainder in practice depends on what you are sharing out. Pens are not normally cut into smaller bits, so a sensible way to express the answer is 7 pens each with 4 pens left over. But money, weights, lengths, areas and volumes can often be subdivided into smaller units. If QR 60 is shared equally among 8 people, then an answer of QR 7.50 makes more sense than QR 7 R QR 4.

In practical situations, results of calculations may have to be rounded up or down in order to make sense of a problem. Calculating 32 ÷ 5 and representing it as 6 R 2 (or 6.4 if done on a calculator) allows discussion about the answer in different contexts.

For example, the 32 might represent a class of students and the 5 the number of students who can sit at a table. If we want to know how many tables are needed to seat the class, we must round up to 7 tables, since 6 tables would not be enough to seat all the students. On the other hand, the 32 might represent QR 32, and the 5 the cost of a book in riyals. If we want to know how many QR 5 books can be bought, we must round down to 6 books, since there is not enough money to buy 7 of them. In this case, the remainder of QR 2 also makes sense.

Towards a standard method of division

For division, as with multiplication, the vast majority of students in Grades 4 to 6 should progress from informal pencil-and-paper methods to a standard written method. At the same time, they need to be reminded to approximate to gain a sense of the size of their answers.

In multiplication, the progression is from informal methods, such as the grid method, to ‘long’ multiplication. The progression allows the teacher to make clear the links between the informal and standard methods in terms of presentation when recording each step in the calculation.

The distributive law for multiplication over addition is recognised and used in the grid method, as in:

27 × 34 Approximate answer: 30 × 30 = 900

×	20	7
30	600	210
4	80	28

810 ← This is 27 × 30
108 ← This is 27 × 4
918 ← This is 27 × 34

Answer: 27 × 34 = 918

From here, it is a short step to an alternative, shorter form of recording, a standard method of long multiplication. This is a goal that can be reached by the vast majority of Grade 5 students and by some students before then.

```
   27
×  34
  810  ←  This is 27 × 30
  108  ←  This is 27 × 4
  918  ←  This is 27 × 34
```

In **division**, the progression is from subtraction of ‘chunks’, often referred to as ‘chunking’, to standard methods for ‘long’ and ‘short’ division.

For example, we can work out how many packs of 24 we can make from 560 biscuits by repeatedly subtracting multiples of 24.

560 ÷ 24 Approximate answer: 550 ÷ 25 = 22

$$\begin{array}{llr}
 & & 560 \\
\text{Make 10 packs} & 24 \times 10 = 240 & -\underline{240} \\
 & & 320 \\
\text{Make 10 packs} & 24 \times 10 = 240 & -\underline{240} \\
 & & 80 \\
\text{Make 2 packs} & 24 \times \ \ 2 = \ \ 48 & -\underline{48} \\
 & & 32 \\
\text{Make 1 pack} & 24 \times \underline{\ \ 1\ } = \ \ 24 & -\underline{24} \\
\text{Total packs} & \quad\ 23 & 8
\end{array}$$

Answer: 23 packs of 24 biscuits, with 8 biscuits remaining

Efficiency is increased if there are fewer steps. The approximation, in this case 550 ÷ 25 = 22, gives a guide to the size of the first chunk. Since the estimate of the answer is 22, the first step might be to subtract 24 × 20.

$$\begin{array}{lr}
 & 560 \\
24 \times \ 20 & -\underline{480} \\
 & 80 \\
24 \times \underline{\ \ 3\ } & -\underline{72} \\
\qquad 23 & 8
\end{array}$$

which can be recorded as:

$$\begin{array}{r|rl}
24 & \overline{)560} & \\
20 & -\underline{480} & \longleftarrow \text{This is } 24 \times 20 \\
 & 80 & \\
3 & -\underline{72} & \longleftarrow \text{This is } 24 \times 3 \\
23 & 8 &
\end{array}$$

and then as:

$$\begin{array}{rr}
 & \underline{\ \ 23} \\
24 &)560 \\
 & -\underline{480} \\
 & 80 \\
 & -\underline{72} \\
 & 8
\end{array}$$

Answer: 23 r 8

The layout on the left is the long division method, although conventionally the 20 or 2 ‘tens’ (represented by 2) and the 3 ‘ones’ forming the answer would be recorded above the line, as on the right. Recording the build-up of the quotient on the left keeps the links with ‘chunking’ and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Examples of the written calculations that students need experience of as far as ‘long’ division is concerned are:

no remainder	21)483	32)736
remainder	33)718	
positioning of the first digit of the quotient	21)126	32)224
decimal dividend	18)57.6	83)7.47

‘Short’ division, the standard method used for dividing by a single digit number, is even more compact than ‘long’ division. It can be introduced as an alternative, more compact recording of the mental method of partitioning, which relies on the distributive law. For example, for 81 ÷ 3, the dividend of 81 is split into the greatest multiple of 3 tens or 30, plus the rest. So 81 is split into 60 + 21, and each part is divided by 3.

$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

The short division process is represented like this:

$$\begin{array}{r} 2\;7 \\ 3\,\overline{)\,8\,{}^{2}1} \end{array}$$

The little 2 represents the 2 tens that have been exchanged for 20 ones. It is written in front of the 1 to show that a total of 21 ones are to be divided by 3. The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.

Short division of a two-digit number can be introduced to students working at level 3 who are confident with multiplication and division facts and with subtracting multiples of 10, and whose understanding of partitioning and place value is sound.

Short division of three-digit numbers is done in stages. For 672 ÷ 4, the dividend of 672 is first split into 400 (the greatest multiple of 4 hundreds or 400), plus the remaining 272.

$$\begin{aligned} 672 \div 4 &= (400 + 272) \div 4 \\ &= (400 \div 4) + (272 \div 4) \\ &= \textcircled{100} + (272 \div 4) \end{aligned}$$

$$\begin{array}{l} \quad\textcircled{1} \\ 4\,\overline{)\,6\,{}^{2}7\;2} \end{array}$$

Then the 272 is split into 240 (the greatest multiple of 4 tens or 40), plus the remaining 32.

$$\begin{aligned} 272 \div 4 &= (240 + 32) \div 4 \\ &= (240 \div 4) + (32 \div 4) \\ &= \textcircled{60} + (32 \div 4) \end{aligned}$$

$$\begin{array}{l} \quad 1\,\textcircled{6} \\ 4\,\overline{)\,6\,{}^{2}7\,{}^{3}2} \end{array}$$

Finally, the 32 is divided by 4, to give 8.

$$32 \div 4 = \textcircled{8}$$

$$\begin{array}{r} 1\;6\,\textcircled{8} \\ 4\,\overline{)\,6\,{}^{2}7\,{}^{3}2} \end{array}$$

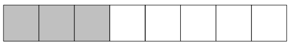
As with column methods of addition and subtraction, the short division process requires careful recording, with the ‘hundreds’, ‘tens’ and ‘ones’ lined up one above the other.

In Grades 5 and 6, students working confidently should be introduced to 'short' division of three-digit numbers. Examples of the calculations that students at levels 4 and 5 need to experience are:

no exchange, no remainder	4)848	
no exchange, with remainder	3)635	
with exchange, no remainder	7)994	
with exchange, with remainder	3)470	
placing of the first digit of the quotient	7)287	
noughts in the quotient	4)816	8)5608
decimal dividend	5)61.5	3)4.26

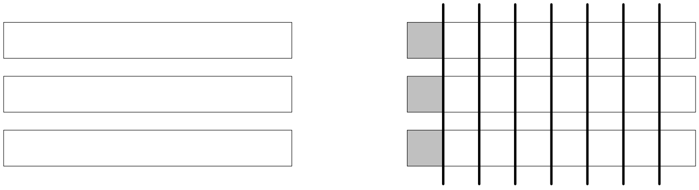
Links between division and fractions

A fraction like $\frac{3}{8}$ is often introduced by dividing one whole shape into eight equal pieces and shading three of them.



$\frac{3}{8}$ of 1

The division 3 ÷ 8, or 3 divided by 8, can be shown to be equivalent to the fraction $\frac{3}{8}$. This calculation might represent, for example, the result of dividing three whole bars of chocolate into eight equal chunks.



3 $\frac{1}{8}$ of 3

The three whole bars of chocolate have been divided into eight equal pieces by slicing across them. As there are three whole bars, each person will get three pieces. We can see from the diagrams that $\frac{1}{8}$ of 3, or 3 divided by 8, is equivalent to $\frac{3}{8}$ of 1.

Focusing on **divisors that are multiples of the dividend** links to work on equivalent fractions. For example:

10 ÷ 20 10 ÷ 30 10 ÷ 40 …

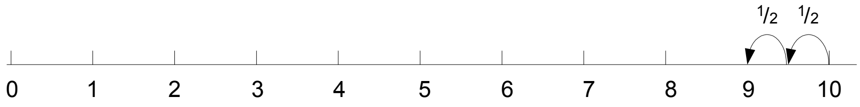
The calculation 10 ÷ 20 is interpreted as: 'Start at 10. How many steps of 20 are needed to get back to zero?' We immediately realise we can only take half a step to reach zero, so that 10 ÷ 20 = $\frac{1}{2}$. An alternative model is to take 10 whole bars of chocolate, each of 20 small squares. When we find $\frac{10}{20}$ by dividing the 10 bars into 20 equal pieces, we see that the result is equivalent to 10 small squares, or $\frac{1}{2}$ of one whole bar.

Let us turn to **divisors between 0 and 1**, such as:

10 ÷ $\frac{1}{2}$ 10 ÷ $\frac{1}{3}$ 10 ÷ $\frac{1}{4}$ … or: 10 ÷ 0.1 10 ÷ 0.2 …

The number line and the 'stepping back to zero' approach is again a helpful model. But the 'bars of chocolate' model falters if we read 10 ÷ $\frac{1}{2}$ as '10 divided between

$^1/_2$' since there is no context that can represent 'sharing 10 between one half'. However, if we read $10 \div {}^1/_2$ as '10 divided into halves', a 'bars of chocolate' image shows that the answer is 20.



On a number line, we can interpret $10 \div {}^1/_2$ as: 'How many halves make 10?' or: 'Start at 10. How many steps of $^1/_2$ are needed to get back to zero?' In this case, we must take two steps each time we move between two consecutive whole numbers. For $10 \div {}^1/_3$, we must take three steps, and so on. So:

$10 \div {}^1/_2 = 10 \times 2 = 20$
$10 \div {}^1/_3 = 10 \times 3 = 30$
$10 \div {}^1/_4 = 10 \times 4 = 40$ …

Expressing quotients as decimals

When they recognise the equivalence of $^{38}/_5$ or $7^3/_5$ and $38 \div 5$, students can use their knowledge of the equivalence between fractions and decimals to express the quotient $7^3/_5$ as the equivalent decimal 7.6.

In the work that they have done with number lines and remainders, students will realise that when they divide a whole number n by a whole number greater than n, the answer is always 0 with a remainder of n. Realising that a division such as $10 \div 15$ is 0 r 10, so that it starts with 'zero point something', is a key step in understanding quotients as decimal fractions.

Students' understanding of quotients as decimal fractions is reinforced by the work that they do with calculators. Students who work out a calculation such as $155 \div 8$ using a mental or written method will get the answer 19 r 3 or $19^3/_8$. Using a calculator, they get the answer 19.375.

Obviously, the 0.375 must be equivalent to $^3/_8$, the fractional part of the quotient $19^3/_8$, but how do you find the remainder of 3 if the calculation $155 \div 8$ has been done on a calculator? We could use the inverse and think of 155 as $(19 + 0.375) \times 8$. To get the remainder, we must multiply the part of the quotient after the decimal point (in this case 0.375) by the divisor (in this case 8), when we get $0.375 \times 8 = 3$.

Multiplying and dividing a decimal fraction by a whole number

The multiplication of a decimal number by a whole number is most likely to occur in the context of money or measures, for example, when we want to find the cost of 17 reels of cotton at QR 2.25 a reel. This problem is modelled by the multiplication 2.25×17. Using a calculator, we would read off the answer 38.25 and interpret this as a total cost of QR 38.25.

Without a calculator, there is the potential for error in the positioning of the decimal point. If students lack confidence with decimals, it is sometimes possible to avoid multiplying the decimal number. In this case, for example, it is possible to change the QR 2.25 to 225 dirhams, then find 225×17 using a written method, to give 3825. We can then change the answer of 3825 dirhams back to riyals, giving a final answer of QR 38.25.

The same advice applies to division of decimals. To cut 4 equal lengths from 6.4 metres of fabric, we can change the 6.4 metres to 640 cm, and then divide by 4 using whatever division method is appropriate. We can change the answer of 160 cm back to 1.6 metres to obtain the final answer.

To deal confidently with these kinds of calculations, students need to be competent at converting one metric unit to another.

Multiplying and dividing by a decimal fraction

Sometimes decimals cannot be dealt with in the way described above. In this case, a calculator is normally used to multiply or divide them, except in simple cases.

For the simple cases, students need an awareness that the smaller the divisor, the larger the quotient. Notice the pattern when, for example, 12 is divided by 3, 0.3, 0.03, 0.003, and so on.

$$\begin{aligned} 12 \div 3 &= 4 \\ 12 \div 0.3 &= 40 \\ 12 \div 0.03 &= 400 \\ 12 \div 0.003 &= 4000 \ldots \end{aligned}$$

Each time the divisor gets 10 times smaller, the quotient gets 10 times bigger. If you think of $A \div B$ as 'A divided into Bs' or 'How many Bs make A?', it is obvious that the smaller that B is, the more times you will be able to subtract it from A.

It is also true that the larger the divisor, the smaller the quotient. Each time the divisor gets 10 times larger, the quotient gets 10 times smaller.

$$\begin{aligned} 12 \div 3 &= 4 \\ 12 \div 30 &= 0.4 \\ 12 \div 300 &= 0.04 \\ 12 \div 3000 &= 0.004 \ldots \end{aligned}$$

Once these principles are understood, it is possible to work out quickly the answer to calculations such as:

$$36 \div 0.003 \qquad \text{or} \qquad 36 \div 3000$$

We can start with the simplest case and then work towards what is needed.

$$\begin{aligned} 36 \div 3 &= 12 & \qquad 36 \div 3 &= 12 \\ 36 \div 0.3 &= 120 & \qquad 36 \div 30 &= 1.2 \\ 36 \div 0.03 &= 1200 & \qquad 36 \div 300 &= 0.12 \\ 36 \div 0.003 &= 12\,000 & \qquad 36 \div 3000 &= 0.012 \end{aligned}$$

Summary

- Students need to know multiplication and division facts 'by heart' before they go on to multiplying and dividing bigger numbers.
- Give students plenty of opportunities to express a multiplication fact such as $12 \times 8 = 96$ as $12 = 96 \div 8$, and $8 \times 12 = 96$ as $8 = 96 \div 12$.
- Teach students how to multiply multiples of 10 before they go on to multiply by two-digit numbers: for example, how to find 30×5, 3×50, and 30×50.

- Before they multiply three-digit numbers, students need to be able to multiply multiples of 100: for example, how to find 300 × 5, 300 × 50 and 300 × 500.
- To become proficient at ‘chunking’, students need to be able to multiply and subtract rapidly, confidently and accurately.
- Teach students two mental calculation strategies for multiplying and dividing two-digit numbers by a single-digit number: factorising and partitioning.
- Before all but the most simple multiplication and division calculations, students should approximate first to gain a sense of the size of the answer and to identify efficient ‘chunks’ to subtract when dividing.
- The majority of students in Grades 4 to 6 should progress from using informal pencil-and-paper methods for multiplication and division to using and applying an efficient standard written method. Students working confidently should be able to do ‘short’ division.
- When the divisor does not divide the dividend exactly, the quotient can be expressed as a whole number with a whole-number remainder, a mixed number or a decimal fraction. How to express the answer to a particular division problem will depend on the context.
- The division $3 \div 8$ is equivalent to the fraction $^3\!/_8$.
- In money and measurement contexts, help children who lack confidence with decimals to avoid multiplying or dividing a decimal quantity by changing it into a whole number of units. After the calculation has been done, and if it is appropriate to do so, the units can be changed back again.
- Students will normally use a calculator for multiplications and divisions involving more than three-digit by two-digit whole numbers, and for multiplication and division by decimals.
- Students need to be aware that the bigger the divisor, the smaller the quotient; the smaller the divisor, the bigger the quotient.
- Help students to ‘see’ a division by relating it back to the number line so that they understand why and how a method works.

Self-assessments

- You can calculate 36 × 15 by factorising 15 and writing the product as 36 × (5 × 3). Show how you would record the use of the associative law.
- You can calculate 28 × 7 by partitioning 28 and writing the product as (20 + 8) × 7. Show how you would record the use of the distributive law.
- How would you partition 91 if you wanted to calculate 91 ÷ 7 by using the distributive law?
- Use the distributive law, and the fact that 221 = 130 + 91, to work out the answer to 221 ÷ 13. Now work it out using 221 = 260 – 39.
- How does the commutative law help you to work out 3 × 1420?
- How would you explain to students a written method for dividing 972 by 36?
- How would you expect students to read the calculation $8 \div {}^1\!/_3$? What kind of model or image would you use to explain the calculation?
- Using the fact that 48 ÷ 8 = 6, how would you help students to work towards 48 ÷ 0.008, and 48 ÷ 80 000?