

Six lessons on division

Lessons in this section

- 1 Doubling and halving
- 2 Multiplication and division facts and $TU \times U$
- 3 Multiplication and division as inverse operations
- 4 $HTU \div U$ (whole-number answers)
- 5 Expressing a quotient as a fraction or decimal
- 6 Tests of divisibility

Resource sheets for the lessons

Using these lesson plans

Choose lessons that will provide a suitable degree of challenge for the students that you will teach. In general, the lessons are appropriate for these grades, depending on students' experience of division and whether the lesson is revision or new work:

Grade 3: Lessons 1 and 2

Grade 4: Lessons 2, 3 and 4

Grade 5: Lessons 4 and 5

Grade 6: Lessons 5 and 6

Each lesson plan has sufficient material to support 40 to 45 minutes of direct teaching. You will need to add suitable practice material for students to incorporate into the lesson, choosing from exercises or activities in published schemes or your own materials. The practice can be offered at any point of the lesson, especially between the 'episodes' that form the main teaching activity. Different tasks can be given to different groups of students, according to their needs.

Alternatively, you could supplement the main part of a lesson by extending your questioning of students or increasing the number of examples that you demonstrate.

Each lesson plan lists the relevant vocabulary and resources. Some of the lessons make use of interactive teaching programs (ITPs). These are animated teaching aids that can be used with a computer and data projector or an interactive whiteboard (download these programs free from www.standards.dfes.gov.uk/primary/publications/mathematics/itps/).

1

Doubling and halving

Objectives

- Recall multiplication facts to 10×10 and associated division facts.
- Double and halve two- and three-digit numbers.

Starter

Vocabulary

multiplied by
divided by
product
multiple
remainder

Resources

counting stick

As a class, recite the 2 times table, forwards and backwards. Ask a few random questions, varying the wording. Make sure that students know the term *multiple*.

Q What is 8 multiplied by 2?

Q What is 18 divided by 2?

Q What is the next multiple of 2 after 14?

Q What is the remainder when 13 is divided by 2?

Explain that 20 is the *product* of 2 and 10, or 2×10 .

Q What is the product of 5 and 2? Tell me two other ways of saying this.
(5 multiplied by 2 and 5 times 2)

Use a counting stick.



Tell students that one end is zero, and the other end is 20. Count along the stick and back again in 2s. Point randomly at divisions on the stick, saying:

Q What is this number? How do you know?

Encourage students to use ‘multiplied by’ and ‘divided by’ in their answers. Point out that they can use the midpoint of the stick as a reference point, for example: ‘I know that halfway is 20 divided by 2, or 10, and the next point is 2 more, or 12.’

Now ask the class to count along the stick and back again in 20s: ‘zero, twenty, forty, ...’ Once again, point at divisions on the stick, saying:

Q What is this number? How do you know?

Use the counting stick to practise chanting the division table for 2, pointing to divisions on the stick as you do so: ‘zero divided by 2 is 0, 2 divided by 2 is 1, 4 divided by 2 is 2, 6 divided by 2 is 3, ...’ and so on.

Main activity

Vocabulary

double
halve

Resources

none

Practise doubling some numbers up to 10: double 3, double 9, double 7. Remind students that doubling is the same as multiplying by 2. Say that double 40 is the same as double 4 multiplied by 10, and extend to doubling multiples of 10: double 30, double 80, double 60.

Write on the board a selection of whole numbers under 50:

17 19 21 24 28 32 35 38 43 46

Ask students if they can double any of the numbers straight away (e.g. 21, 32). Cross out these numbers and record on the board, for example, $21 \times 2 = 42$, $32 \times 2 = 64$.

Ask students to use their exercise books to double the remaining numbers. Allow a couple of minutes, then go through the numbers one by one, inviting students to the board to explain their method to the class. Look for these methods:

- using known facts,
e.g. 19×2 is 2 less than double 20;
- splitting the number into tens and ones or units,
e.g. 28×2 is double 20 + double 8;
- splitting the number in other ways,
e.g. 38×2 is double 35 plus double 3.

Use a diagram to show students how they can always double a two-digit number by doubling the tens and doubling the ones or units.

$$\begin{array}{r}
 47 \\
 40 + 7 \\
 \downarrow \quad \downarrow \quad \times 2 \\
 80 + 14 = 94
 \end{array}$$

Ask students to use this method to double 37, then 78, doing as much as possible mentally.

Repeat the above for halving numbers, starting with some simple practice of halving numbers to 20, including odd numbers. Check that students can express an answer to half of 15 both as a mixed number (seven and a half or $7\frac{1}{2}$) and as a decimal (seven point five or 7.5).

Write $14 \div 2 = 7$ on the board, and $140 \div 2 = 70$. Explain that:

$$140 \div 2 = (10 \times 14) \div 2 = 10 \times (14 \div 2) = 10 \times 7 = 70$$

Stress that the answer is the same as 10 times $14 \div 2$. Practise halving a few more multiples of 10 to 200, and multiples of 100 to 2000.

Give the class some two-digit numbers under 100 to halve, inviting them to explain their strategies. Show them how they can always halve two-digit numbers by partitioning into tens and ones or units, and how to halve sums of money (with an even number of pence) by partitioning into pounds and pence, using diagrams similar to those for doubling.

$$\begin{array}{r}
 75 \\
 70 + 5 \\
 \downarrow \quad \downarrow \quad \div 2 \\
 35 + 2.5 = 37.5
 \end{array}$$

Other tasks

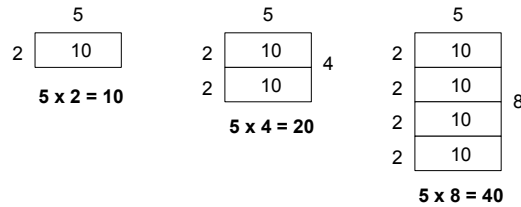
If necessary, choose further related activities from available textbooks or your own materials.

Consolidation

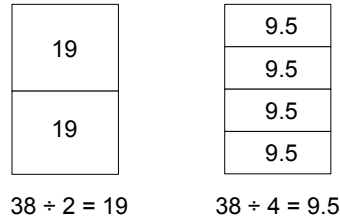
Resources

OHP calculator
OHT 1.1

Use an OHP calculator to demonstrate that any number multiplied by 4 is the same as the number multiplied by 2, then multiplied by 2 again. Demonstrate again by using a diagram to show equivalent areas.



Similarly, use the OHP calculator and then diagrams to show that a number divided by 4 is the same as the number divided by 2, then divided by 2 again.



Show students how to work out the 4 times table from the 2 times table, and the 8 times table from the 4 times table. Complete **OHT 1.1** using doubling to generate the 4 times and 8 times tables, working across the rows. Ask questions such as:

- Q What is 6 multiplied by 2? 6 multiplied by 4? 6 multiplied by 8?**
- Q What is 18 divided by 2? 36 divided by 4? 72 divided by 8?**

Do this initially with the OHT in place, then remove it. Encourage students to use doubling strategies if they cannot remember a fact in the 4 or 8 times tables.

Finish by chanting the division table for 4, pointing to the OHT as you do so: ‘zero divided by 4 is zero, 4 divided by 4 is 1, 8 divided by 4 is 2’, and so on.

Summary for students

- *Double, twice and multiply by 2* mean the same thing.
- Halving is the same as dividing by 2.
- You can multiply a number by 2 by doubling it. You can multiply it by 4 by doubling it again, and by 8 by doubling again.
- To find the *product* of some numbers, you multiply them together.

2

Multiplication and division facts and $TU \times U$

Objectives

- Recall multiplication facts to 10×10 and associated division facts.
- Use an informal written method to calculate $TU \times U$.

Starter

Vocabulary

multiplied by
divided by
product
multiple
remainder

Resources

none

As a class, recite the 8 times table, forwards and backwards. Ask a few random questions, varying the wording.

Q What is 32 divided by 8?

Q What is the product of 5 and 8?

Q What is the next multiple of 8 after 40?

Q How many eights make 56?

Q What is the remainder when 50 is divided by 8?

Discuss ways to remember awkward facts. For example, to remember 10 times a number is always easy. To find 5 times a number is also easy, as it is half of 10 times the number. For example, 10 times 8 is 80, so 5 times 8 is half of 80, or 40.

Remind the class that they can always work out 8 times a number by starting with twice the number, doubling it to get 4 times the number, and doubling again to get 8 times the number.

Main activity

Vocabulary

estimate

Resources

OHT 2.1
counting stick
ITP *Multiplication grid*
(optional)

Use a counting stick and count along it and back again in multiples of 3. Then chant the 3 times table forwards and backwards: ‘one three is three, two threes are six, three threes are nine, ...’.

Show students how they can use doubling to work out the 6 times table from the 3 times table by completing **OHT 2.1**, working across the rows. Ask questions like:

Q What is 5 multiplied by 3? What is 5 multiplied by 6?

Q What are 7 threes? What are 7 sixes?

Q How many threes make 24? How many sixes make 24?

Do this first with the OHT in place. Use the counting stick to practise counting along it and back again in multiples of 6. Remove the OHT and continue the questioning. Remind students to use their knowledge of the 3 times table and doubling strategies if they cannot remember a fact in the 6 times table.

Remind students of the commutative law of multiplication: seven twos are the same as two sevens. Remind them that if they know a multiplication fact one way round, they know it the other way round as well.

Build up the 7 times table on the board, or add it to the OHT, using facts that students already know. Chant the table, forwards and backwards.

Remind the class that they know how to multiply a number by 10 or 100, and that the digits will move one or two places to the left accordingly.

Q How could we multiply a number by 20?

Demonstrate that $7 \times 20 = 7 \times 2 \times 10 = 14 \times 10 = 140$.

Q How could we multiply a number by 60?

Demonstrate that $8 \times 60 = 8 \times 6 \times 10 = 48 \times 10 = 480$.

Q How could we multiply a number by 600?

Demonstrate that $9 \times 600 = 9 \times 6 \times 100 = 54 \times 100 = 540$.

You could, if you wish, use the interactive teaching program *Multiplication grid* (www.standards.dfes.gov.uk/primary/publications/mathematics/itps/) to introduce students to the grid method of multiplication for $TU \times U$ in this part of the lesson. Select options and ask questions similar to those below.

Write $7 \times 53 = 53 \times 7$ on the board.

Q How can we use our knowledge of multiplication to get an estimate of 7×53 ?

Establish that the answer will lie between $7 \times 50 = 7 \times 5 \times 10 = 350$ and $7 \times 60 = 7 \times 6 \times 10 = 420$. It will be closer to 350 than to 420, since 53 is closer to 50 than to 60.

Point out that 53 can be written as $50 + 3$. Draw a grid on the board.

x	50	3
7		

x	50	3
7	350	21

Work through the left-hand grid with the class to get the right-hand grid. Ask:

Q How can we get the answer to 53×7 from the grid?

Add 350 and 21 mentally to get the answer 371. Work through one or two more examples, such as 37×4 , 72×6 . Explain that the grid can be used as a jotting to support or explain a mental calculation.

Say to students that they may be able to do simple examples mentally without writing anything. Try 13×4 and 16×6 as mental calculations.

Other tasks

Choose further practice and consolidation material, selecting from available textbooks or your own resources.

Consolidation

Vocabulary

divided exactly by

Resources

100-square or OHT 2.2
mini-whiteboards

Show **OHT 2.2**, a 100-square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Ask students to use their whiteboards and to answer questions such as these.

Q I am thinking of a multiple of 8 lying between 50 and 60. What is it? (56)

Q I am thinking of a number lying between 30 and 40 that can be divided exactly by 9. What is it? (36)

Q I am thinking of a multiple of 7 that is greater than 40 and less than 50. What could it be? (42, 49)

Q I am thinking of a multiple of 5 that can be divided exactly by 6. What could it be? (30, 60, 90, ...)

Invite individual students to justify their answers by asking:

Q How do you know?

Expect them to justify by stating the relevant multiplication or division fact: 'I know that 56 is a multiple of 8 because $7 \times 8 = 56$ and $56 \div 8 = 7$.'

Summary for students

- Use multiplication facts that you know to work out other multiplication and division facts.

3

Multiplication and division as inverse operations

Objectives

- Identify and use the inverse relationship between multiplication and division
- Use a calculator effectively

Starter

Vocabulary

factor
multiple
product

Resources

mini-whiteboards

Write $8 \times 5 = \square$ on the board.

Establish that the answer is 40 and place it in the box. Explain that this number sentence is part of a family of four.

Q What are the other associated number sentences?

Collect the other three number sentences: $5 \times 8 = 40$, $40 \div 8 = 5$, $40 \div 5 = 8$.

Reinforce that for each number sentence there are usually three others. Work through $5 \times 5 = 25$ to show that in this case there is only one associated number sentence.

Remind the class that 40 is a multiple of 8 and a multiple of 5, and that both 8 and 5 are factors of 40.

Write $9 \times 4 = \square$ on the board. Ask students to use whiteboards to show an associated number sentence. Check for all three possibilities from the responses.

Write $42 \div 6 = 7$ on the board and obtain the other three number sentences from the class.

Q What is the product of 6 and 7?

Draw out the meaning of the word 'product'.

Q How can we describe the connection between 6, 7 and 42 using the words multiple and factor?

Highlight that 6 is a factor of 42 and 42 is a multiple of 7. When discussing these, refer to the appropriate number sentence and emphasise the vocabulary being used.

Main activity

Vocabulary

commutative
inverse operations

Resources

mini-whiteboards
calculators
OHP calculator
OHT 3.1

Show the upper half of OHT 3.1.

Q How many counters are there in each cell?

Q How many cells are there?

Q How many counters are there altogether?

Q How can we represent this as a number statement?

Collect responses and write $6 \times 8 = 48$ on the board. Remind the class again that each number sentence is usually a member of a family of four.

Write $\square \div 8 = 6$ on the board, and ask the class for the other three number sentences.

$$6 \times 8 = \square \quad 8 \times 6 = \square \quad \square \div 6 = 8$$

Q Which number sentences can you complete?

Reinforce that knowing 6 multiplied by 8 is 48, means we know that 48 divided by 8 must be 6. Ensure that students can complete all the sentences.

Show the lower half of **OHT 3.1** and ask the same four questions.

- Q How many counters are there in each cell?**
- Q How many cells are there?**
- Q How many counters are there altogether?**
- Q How can we represent this as a number statement?**

Write $\square \div 4 = 8$ on the board.

- Q What number goes in the empty box?**

Using **OHT 3.1**, emphasise that there are 8 fours or 4 eights, and $8 \times 4 = 32$. So 32 must go in the box. Ask the class to record the other three number sentences on their whiteboards, using the empty box as before. Write on the board:

$$4 \times 8 = \square \quad 8 \times 4 = \square \quad \square \div 8 = 4$$

- Q Which number sentences can we complete?**

Establish that knowing the multiplication fact $8 \times 4 = 32$ or $4 \times 8 = 32$ is enough. We can use this fact to fill in all boxes. Say that multiplication and division are *inverse operations*. The inverse operation for division is multiplication, and the inverse operation for multiplication is division.

Write $\square \div 6.3 = 4$ on the board.

- Q What number goes in the empty box?**

Emphasise that there are four 6.3s, so 6.3×4 must go in the box. Work this out using:

$$6 \times 4 = 24 \text{ and } 0.3 \times 4 = 1.2, \text{ so that } 6.3 \times 4 = 25.2.$$

Reinforce with a diagram.

$$\begin{array}{r}
 6.3 \\
 6 \quad + \quad 0.3 \\
 \downarrow \quad \quad \downarrow \quad \times 4 \\
 24 \quad + \quad 1.2 \quad = \quad 25.2
 \end{array}$$

x	6	0.3
4		

x	6	0.3
4	24	1.2

Repeat with several more examples, such as $\square \div 2.7 = 8$, and $\square \div 5.9 = 9$

Other tasks

Choose further practice and consolidation material, selecting from available textbooks or your own resources.

Consolidation

Resources

mini-whiteboards
calculators

Write $2.8 \times \square = 10.36$ on the board. Ask the class to use their whiteboards to write down the other three number sentences using the empty box notation.

- Q Which number sentence can be completed to help us to find the number in the box?**

Encourage students to use a calculator to work out $10.36 \div 2.8$.

Q How can we check our answer?

Establish that we can check the answer by putting the answer into the original number sentence. The calculation would be 2.8×3.7 .

Now ask the class to work out the missing number in $17.4 \times \square = 40.02$. Discuss their strategies. Agree that the check calculation is 17.4×2.3 .

Write $\square \div 2.5 = 5.4$ on the board. Establish that this missing number can be worked out by calculating 5.4×2.5 . Relate this to earlier examples. Agree that the check calculation is $13.5 \div 2.5$.

Now ask the class to work out the missing number in $\square \div 3.8 = 7.6$. Discuss their strategies. Confirm that the check calculation is $28.88 \div 3.8$.

Summary for students

- When finding a missing number, it is helpful to write down the other three number sentences and then decide which one to use to find the missing number.
- Always put your answer back in the question and check that it works.

4

HTU ÷ U (whole-number answers)

Objectives

- Recall multiplication facts to 10×10 and derive associated division facts.
- Calculate mentally $TU \div U$.
- Use a written method to calculate $HTU \div U$.
- Check whether a result is the right order of magnitude.
- Round up or down after division, depending on context.

Starter

Vocabulary

times
multiplied by
divided by
product
multiple
remainder
squared

Resources

mini-whiteboards

As a class, chant the 3 times table, forwards and backwards. Ask:

Q Which statements in the 3 times table are easy to remember? Why? Which are harder to remember? (e.g. 3×7 , 3×8)

Repeat the 'harder' facts three times each, and the associated division facts.

Ask questions related to the table, varying the wording, getting students to answer using their whiteboards. For example:

Q What is 9 times 3?

Q What is 27 divided by 3?

Q What is the product of 6 and 3?

Q What is 3 squared?

Q What is 3 multiplied by 8?

Q What is the next multiple of 3 after 30? How did you work it out? (add 3 to 30)

Q How many threes are there in 21?

Q What is the remainder when 29 is divided by 3?

Main activity

Vocabulary

problem
calculation
division
factors
rounded up/down

Resources

Resource 4.1
ITP *Grouping* (optional)

You could, if you wish, introduce the main activity by using the program *Grouping* (www.standards.dfes.gov.uk/primary/publications/mathematics/itps/).

Write $42 \div 3$ on the board. Show the class how to partition to calculate this mentally. Split the first number into a multiple of 10 that is an exact multiple of the divisor 3, plus the rest. So 42 is split into $30 + 12$, then each part is divided by 3.

$$\begin{array}{r} 42 \\ 30 + 12 \\ \downarrow \quad \downarrow \quad \div 3 \\ 10 + 4 = 14 \end{array}$$

Practise some examples, such as $68 \div 4$ (splitting 68 into $40 + 28$) and $90 \div 6$ (splitting 90 into $60 + 30$).

Write this problem on the board, or prepare it on an OHT.

*There are 6 stamps on every card of self-adhesive stamps.
Darwish bought a total of 138 stamps.
How many cards of stamps did Darwish buy?*

Ask students to discuss with a partner how to answer the problem, then take feedback.

Establish that we need to know how many sixes there are in 138, or $138 \div 6$. Confirm that the answer to this problem will be a whole number bigger than 20 (enough cards for 120 stamps) and less than 30 (enough cards for 180 stamps), but that it will be closer to 20 than to 30 because 138 is closer to 120 than to 180. Illustrate this if necessary by drawing an empty number line, marking the multiples of 10 from 120 to 180, and pointing out the position of 138.

One way to work out the answer would be to keep subtracting 6 stamps but this might take a long time. It would be easier to take away 60 stamps for 10 cards in one go, and then see what is left.

Write on the board:

$$\begin{array}{r}
 138 \\
 - \underline{60} \quad 6 \times 10 \\
 78 \\
 - \underline{60} \quad 6 \times 10 \\
 18 \\
 - \underline{18} \quad 6 \times 3 \\
 0
 \end{array}$$

Q How many lots of 6 stamps have we taken away? ($10 + 10 + 3 = 23$)

Establish that the answer to $138 \div 6$ is 23, so that Darwish bought 23 cards.

Show students how the calculation could be made more efficient by subtracting 120 stamps all at once.

$$\begin{array}{r}
 138 \div 6 \\
 138 \\
 - \underline{120} \quad 6 \times 20 \\
 18 \\
 - \underline{18} \quad 6 \times 3 \\
 0
 \end{array}$$

Demonstrate an example with a remainder, such as $140 \div 6$, recording the answer as 23 R 2. Discuss with the class how to deal with the remainder in the context of two different problems.

*A baker packs cakes in boxes of 6.
The baker has baked 140 cakes.
How many boxes can the baker fill?*

*Eggs are packed in boxes of 6.
Some hens have laid 140 eggs.
How many boxes will be used to pack all the eggs?*

Explain that in the first problem the answer will be a whole number of boxes. $140 \div 6$ is 23 R 2. The answer will need to be rounded down to 23, the number of full boxes of cakes. In the second problem, the answer will again be a whole

number of boxes, but in this case the answer must be rounded up to 24 because 23 of the boxes will hold 6 eggs, and one box will hold 2 eggs. Stress to students that they need to think about the context of the problem before they round up or down.

Give out **Resource 4.1**, and work through the problems with the class. Ask questions such as:

- Q What is the question asking us to calculate?**
- Q What are the key words in the question?**
- Q Which operation shall we use?**
- Q What is the approximate answer?**
- Q How shall we show our working?**
- Q Should the answer be rounded up or down in the context of the question?**

Make sure that students can explain the key steps in the calculation. Demonstrate how they should set out their work.

Other tasks

Choose further practice and consolidation material, selecting from available textbooks or your own resources.

Consolidation

Resources

OHT 4.2

Show **OHT 4.2**, and complete with the class the first multiplication table.

Show the second multiplication table and ask students to study it in pairs. Tell them that the task is to fill in all the blank spaces in the table. If they have difficulty in getting started, suggest that they think about the possible factors of 21 in the bottom right corner of the table.

After a few minutes, take feedback, and work through the solution. Emphasise that using knowledge of multiplication and division facts helped to eliminate or confirm possible values, and that to solve the whole puzzle required systematic working.

Summary for students

- To work out mentally the answer to a calculation such as $56 \div 4$, split the first number into a multiple of 10 that is an exact multiple of the divisor, plus the rest. So 56 is split into $40 + 16$, then each part is divided by 4.
- When you calculate $HTU \div U$, estimate the answer first. Set the calculation out carefully and work systematically and efficiently. Check the answer against the estimate.
- Check that the answer to a division calculation makes sense in the context of the problem. Think carefully whether it should be rounded up or down.

5

Expressing a quotient as a fraction or decimal

Objectives

- Recall multiplication facts to 10×10 and associated division facts.
- Calculate mentally $TU \times U$ and $TU \div U$.
- Express a quotient in fraction or decimal form.
- Divide a decimal such as 6.28 by a single-digit number.

Starter

Vocabulary

divided by

Resources

digit cards or
mini-whiteboards

As a class, chant the 6 times table, forwards and backwards.

Tell students that it is often possible to work out a calculation like 17×6 in their heads. Show them how to split the larger number and to jot down each part. Record on the board:

$$\begin{array}{r} 17 \\ 10 + 7 \\ \downarrow \quad \downarrow \\ 60 + 42 = 102 \end{array} \times 6$$

Explain that this can be written as $17 \times 6 = (10 + 7) \times 6 = 60 + 42 = 102$. Practise a couple of example, such as 18×5 and 24×3 .

Extend to decimals, with a calculation like 1.4×7 .

$$\begin{array}{r} 1.4 \\ 1 + 0.4 \\ \downarrow \quad \downarrow \\ 7 + 2.8 = 9.8 \end{array} \times 7 \quad \text{and } 1.4 \times 7 = (1 + 0.4) \times 7 = 7 + 2.8 = 9.8.$$

Practise one or two more examples, such as 2.3×5 and 3.4×6 .

Explain that a calculation like $91 \div 7$ can be done similarly. However, the first number has to be split in a slightly different way, into the largest multiple of 10 that is an exact multiple of the divisor 7, plus the rest. So 91 is split into $70 + 21$.

$$\begin{array}{r} 91 \\ 70 + 21 \\ \downarrow \quad \downarrow \\ 10 + 3 = 13 \end{array} \div 7 \quad \text{and } 91 \div 7 = (70 + 21) \div 7 = 10 + 3 = 13.$$

Practise some examples, such as $85 \div 5$ (splitting 85 into $50 + 35$) and $78 \div 3$ (splitting 78 into $60 + 18$).

Main activity

Vocabulary

remainder

Resources

digit cards or
mini-whiteboards

Say that sometimes one number will not divide exactly into another. The remainder is what is left over.

Q What is the remainder when 26 is divided by 6?

Q How do you know that you are right?

Remind students that if they are asked to explain how they know they are right, it is helpful to describe or write a calculation. Establish that $26 = (6 \times 4) + 2$, and that 2 is the remainder.

Tell the class that you are going to call out some numbers. You want them to use their digit cards or whiteboards to show what the remainder would be when that number is divided by 6. Call out whole numbers less than 60, and check responses. Each time, ask:

Q How do you know that you are right?

Write on the board $21 \div 5$. Establish that this is 4 R 1, and complete:

$$21 \div 5 \text{ is } 4 \text{ R } 1$$

Draw an empty number line on the board. Demonstrate jumping on in steps of 5 to 20, then count the jumps: one, two, three, four. Say that four fives jump to 20. Establish that five fives would jump to 25 and would be too much. The answer to $21 \div 5$ must be greater than 4 but less than 5. To get to 21 would need four and a bit fives.

Write on the board $21 = 20 + 1$. Point to the 20 and say: '20 divided by 5 equals 4'. Point to the 1, and ask:

Q How could we represent 1 divided by 5?

Establish that $1 \div 5$ is $\frac{1}{5}$. Say that $21 \div 5 = 4\frac{1}{5}$. Record:

$$21 \div 5 \text{ is } 4 \text{ r } 1 \text{ or } 4\frac{1}{5}.$$

Repeat the above for $23 \div 5$ and $24 \div 5$, and record in the same way:

$$23 \div 5 \text{ is } 4 \text{ r } 3 \text{ or } 4\frac{3}{5} \quad 24 \div 5 \text{ is } 4 \text{ r } 4 \text{ or } 4\frac{4}{5}$$

Q What do you think the answer to $22 \div 5$ will be?

Establish that the answer to $22 \div 5$ is 4 r 2 or $4\frac{2}{5}$.

Q How can we write $\frac{2}{5}$ as a decimal? (0.4)

Remind students that $4\frac{2}{5}$ can be written as $4 + 0.4 = 4.4$. Repeat for $4\frac{3}{5}$ and $4\frac{4}{5}$.

Ask the class:

Q QR27 is shared equally among 6 people. How much does each person get?

Establish that the answer to $27 \div 6$ is $4\frac{3}{6}$ or 4.5.

Q What amount of money does this represent? (QR 4.50)

Write on the board $63 \div 10$. Establish that the answer is 6 r 3. Use an empty number line to demonstrate that the answer will be greater than 6 but less than 7.

$$\text{Write } 63 \div 10 = (60 + 3) \div 10 = (60 \div 10) + (3 \div 10).$$

Point to $60 \div 10$, and ask for the answer (6). Point to $3 \div 10$, and ask for the answer (three tenths or 0.3).

Complete $63 \div 10 = (60 \div 10) + (3 \div 10) = 6 + \frac{3}{10} = 6\frac{3}{10}$.

Q How can we write $6\frac{3}{10}$ as a decimal?

Establish that $6\frac{3}{10} = 6.3$.

Q How would you explain your answer to $94 \div 10$?

Establish that $94 \div 10$ can be written as $(90 \div 10) + (4 \div 10) = 9 + \frac{4}{10}$. Stress that the remainder of 4 is divided by 10 to give $\frac{4}{10}$.

Q How can we write $9\frac{4}{10}$ as a decimal?

Establish that $9\frac{4}{10} = 9.4$.

Ask students to choose whole numbers to divide by 10, writing their answers in both fraction and decimal form. Take feedback and discuss methods.

Other tasks

Choose further practice and consolidation material, selecting from available textbooks or your own resources.

Consolidation

Remind the class that, when dividing by 10, the remainder is divided by 10, and can be written as a fraction in tenths. When dividing by 5, the remainder is divided by 5, and can be written as a fraction in fifths. In each case there are decimal equivalents.

Q What is the answer to $13 \div 2$?

Establish that answer is 6 r 1. The remainder is 1, which will be divided by 2, and then written as $\frac{1}{2}$, so that the answer to the calculation is $6\frac{1}{2}$ or 6.5.

Q What is the answer to $33 \div 4$?

Establish that the answer is 8 r 1, and that the remainder of 1 will be divided by 4. This can be written as $\frac{1}{4}$ or its decimal equivalent of 0.25, to make an answer of $8\frac{1}{4}$ or 8.25. Refer to the number line if necessary.

Ask the class to divide 19 by 2, 4, 5 and 10. Record the answers, in remainder form, and in fraction and decimal equivalents.

Summary for students

- When dividing by 10, the remainder will be tenths. A remainder of 7 can be represented as $\frac{7}{10}$ or 0.7.
- When dividing by 2, any remainder is 1, represented as $\frac{1}{2}$ or 0.5.
- When dividing by 4, the remainder will be quarters. A remainder of 1 can be represented as $\frac{1}{4}$ or 0.25, and of 3 as $\frac{3}{4}$ or 0.75.

6

Tests of divisibility

Objectives

- Recall multiplication facts to 10×10 and derive associated division facts.
- Recognise multiples and use tests of divisibility.
- Solve mathematical problems, recognise and explain patterns and relationships, generalise and predict.

Starter

Vocabulary

product
sum

Resources

none

Write four single-digit numbers on the board: for example, 3, 5, 4, 8. Tell the class that you want to make two pairs of numbers. Multiply the numbers in each pair, then add the two products together. Record on the board:

$$\begin{array}{cccc}
 3 & 5 & 4 & 8 \\
 3 \times 5 & + & 4 \times 8 & \\
 \downarrow & & \downarrow & \\
 15 & + & 32 & \\
 \swarrow & & \searrow & \\
 & 47 & &
 \end{array}$$

Q Is 47 the biggest answer we can get? Can we pair the numbers to get a larger answer?

Ask students to discuss this in pairs, trying out different combinations of numbers. They should find that pairing 3, 4 and 5, 8 results in a larger answer: 52. Give them four more single-digit numbers to make the biggest possible answer in a similar way. Confirm the answer.

Now ask students to choose and use four numbers of their own.

Q Can you find any rules for how to pair off the four numbers to give you the largest possible result?

Allow time for further investigation, then discuss findings. Draw out that pairing the largest two numbers and the smallest two numbers gives the largest result. Ask:

Q What if we add the numbers in each pair, and then multiply the two sums? How should we pair the numbers to get the biggest result?

After further investigation, draw out that the largest result in this case is produced by pairing the middle two numbers, and the largest and the smallest number.

Main activity

Vocabulary

multiple
factor
divisible
digit sum

Resources

OHP calculator
OHT 6.1

Explain that if you multiply the counting numbers by 7 the table that you would get would start like this.

1×7	2×7	3×7	4×7	5×7	6×7	7×7
7	14	21	28	35	42	49

Remind students that 7, 14, 21, ... are the multiples of 7, and that 7 is a factor of every multiple of 7. The complete set of factors of 21 is 1, 3, 7 and 21.

Show **OHT 6.1**, a set of 'Who am I?' puzzles.

Show students how to solve the first problem by listing numbers systematically. For example, list in column A all the multiples of 4 less than 20. In column B, add

3 to each number to create numbers that have a remainder of 3 when divided by 4. In column C, list the multiples of 5 less than 20. In column D, add 4 to each of these numbers to give numbers that have a remainder of 4 when divided by 5. Now look for the number that is common to both columns B and D.

A	B	C	D
4	7	5	9
8	11	10	14
12	15	15	19
16	19		

Ask students to work in pairs to solve the other three puzzles. Remind them about working systematically. When time is up, ask pairs of students to present their solutions to the class, drawing out the answers to questions 2, 3 and 4: 63, 99 and 20, 83 and 36, 81 respectively.

Write 456 on the board. Choose a student to read out the number in words.

Q Is this number odd or even? (even)

Point out that the last digit, 6, is even, so the whole number is even. Explain that an even number is *divisible* by 2, which means that it divides exactly by 2 with no remainder. A number that is divisible by 2 is also a multiple of 2.

Say a few numbers and ask students to put up a hand if the number is divisible by 2: for example, one hundred and thirty-four, five hundred and seventy-three, one thousand and eight.

Point to 456 again.

Q Is this number divisible by 3?

Tell the class that there is a quick way to find out by adding all the digits. Write $4 + 5 + 6$ on the board and ask for the answer. Point to the answer 15, explaining that this is the sum of the digits. Because the digit sum is a multiple of 3, the whole number 456 is divisible by 3.

Ask the class to test a few numbers for divisibility by 3, writing them on the board: for example, 234, 113, 559, 957. Confirm their results using an OHP calculator.

Point again to 456.

Q Is this number divisible by 4?

Say that there is an easy way to find out. Explain that any multiple of 100 will always be divisible by 4, since $25 \times 4 = 100$. A number like 456 can be split into 400 and 56. Since 400 is divisible by 4, we need only ascertain whether the last two digits, in this case 56, are divisible by 4.

Confirm that 56 is a multiple of 4.

$$\begin{array}{r}
 56 \\
 40 + 16 \\
 \downarrow \quad \downarrow \quad \div 4 \\
 10 + 4 = 14 \quad \text{and } 56 \div 4 = (40 + 16) \div 4 = 10 + 4 = 14.
 \end{array}$$

Ask the class to test a few numbers for divisibility by 4, for example, 128, 146, 504.

Point again to 456.

Q Is this number divisible by 5 or by 10?

Confirm that it is not, since it does not end in 5 or 0.

Q Is it divisible by 6?

Explain that all multiples of 6 divide exactly by 2 and also by 3. We know that 456 divides exactly by 2 because it is even. We also know that it divides exactly by 3 because the sum of its digits is a multiple of 3. So 456 is divisible by 6.

Ask the class to test 144, 1110 and 2898 for divisibility by 6.

Other tasks

Choose further practice and consolidation material, selecting from available textbooks or your own resources.

Consolidation

Resources

OHT 6.2
mini-whiteboards
OHP calculator

Show **OHT 6.2**, a set of division calculations. Go through them one by one. Ask a student to read out the first calculation: nine hundred and forty-one divided by two.

Q When you divide 941 by 2, is there a remainder? Yes or no?

Ask students to write 'Y' for yes or 'N' for no on their whiteboards. Ask:

Q How do you know without doing the calculation? (941 is an odd number since its last digit is 1, so it is not divisible by 2)

Confirm by checking on the OHP calculator.

Repeat with the other calculations.

- B Yes. 356 does not end in 5 or 0, so it is not divisible by 5 and there is a remainder.
- C Yes. The sum of the digits of 1220 is 5, which is not a multiple of 3, so 1220 is not divisible by 3 and there is a remainder.
- D Yes. Any number that is divisible by 8 must be even. As 783 is odd, it cannot be divisible by 8 and there is a remainder.
- E No. 580 is divisible by 4, since half of 580 is 290, which is even. There is no remainder.
- F Yes. If 226 is divisible by 6, it must be divisible by 3 and by 2. It is divisible by 2, since it is even. The sum of its digits is 10, which is not a multiple of 3, so it cannot be divisible by 3. There is a remainder.

Summary for students

- An odd number cannot be divisible by an even number.
- A number is divisible by 3 if the sum of its digits is a multiple of 3.
- A number is divisible by 4 if the last two digits are divisible by 4.
- A number is divisible by 5 if its last digit is 5 or 0.
- A number is divisible by 6 if it is even and is also divisible by 3.