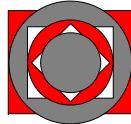


Mathematics workshop 4

Session 1: Problem solving 1



Objectives

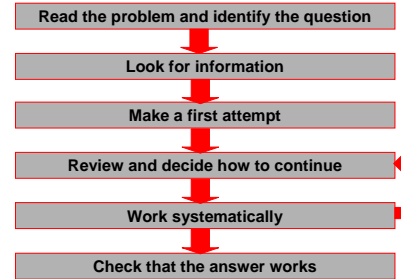


By the end of this session you will:

- know some strategies that students could use to solve mathematical problems
- have considered what 'being systematic' means
- know some approaches to teaching problem solving

1.1

Tackling a problem or puzzle



1.2

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

1.3

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

4 children can sit in a row on a bench by a wall.
In how many different ways can they do it?

1.4

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

The perimeter of a rectangle is 28 cm. The length is
8 cm more than its width. What is its area?

1.5

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

I multiply a number by 3 and subtract 9.
The answer is 24.
What is the number?

1.6

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

What is the sum of the interior angles of a polygon
with n sides?

1.7

Making a first attempt



- Simplify the problem
- Try particular cases
- Make, test and adjust guesses
- Work backwards
- Identify properties that the answer must have

A farmer keeps goats and chickens.
He has 20 of them altogether.
His animals have 54 feet between them.
How many goats does the farmer have?

1.8

Working systematically



- Make lists or tables
- Write out all possibilities and eliminate repetitions
- Fix one variable to see what happens to the others
- Look for, generalise and predict from patterns
- Define rules to limit or structure an activity
- Break a problem into parts

1.9

Tennis game

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Inas	✓	×	×	✓	✓	×	×
Sara	✓	×	✓	✓	×	×	×
Alia	×	✓	✓	×	✓	✓	×
Mai	✓	✓	×	×	✓	×	×

1.10

Tennis game

	Inas	Sara	Alia	Mai
Inas				
Sara	Mon/Thu			
Alia	Fri	Wed		
Mai	Mon/Fri	Mon	Tue/Fri	

1.11

Teaching students to be systematic



- Show students a particular way to be systematic, e.g. making a list or table
- Offer similar problems that can be solved by drawing on the same approach
- Pair students so that a more systematic problem-solver works with one who is less so
- Ask students who have solved a problem systematically to explain their methods to the class
- Display examples of systematic work and refer to them in lessons

1.12

Summary

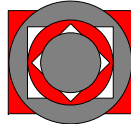


- Teach students how to be systematic – it doesn't happen by accident
- Show students particular strategies and give them related problems to solve
- When they know how to 'be systematic', students have powerful strategies for solving new problems

1.13

Mathematics workshop 4

Session 2: Problem solving 2



Objectives



By the end of this session you will:

- have discussed how students might use reasoning to justify their conjectures and conclusions
- know how a teacher can guide students' reasoning
- know how students might record their reasoning

2.1

What's my rule?



This pair of numbers is connected by a simple rule.

2, 8

Suggest another pair of numbers that satisfies the same rule.

If you think you know the rule, don't say what it is. Just provide further examples to confirm your conjecture.

2.2

Primes



The formula:

$$n^2 - n + 11$$

generates prime numbers for all values of n .

Is this statement true or false?

2.3

Explaining reasoning



- Some questions, including test questions, require students to:
 - 'explain how you know'
 - 'explain why the statement is false'
 - 'explain how this is possible'
- Students need to give a **convincing argument** that explains how or why a particular conclusion has been reached

2.4

The vocabulary of reasoning



- *it could be ..., because ...*
- *it can't be ..., because ...*
- *it won't work, because ...*
- *if ... then ...*
- *it would only work if ...*
- *so ...*
- *in that case ...*

and phrases like: *since, therefore, it follows that ..., it will/won't work when ...*

2.5

Prompts to guide reasoning



- What can you work out (from the information)?
- If you know that, what else do you know?
- Can you tell me what you are thinking?
- Shall we test that?
- Does it work?
- Do you still think it is ... ?
- Do you agree that ... ?
- Why is that bit important?
- So, what must it be?

2.6

Recording known facts

Looked: Number Reasoning

1x	the answer will always be one digit up to nine times
2x	four of the answers are units (single digits)
3x	three of the answers are units (single digits)
4x	two of the answers are units (single digits)

- 1 x The answer will always be one digit up to nine times
- 2 x Four of the answers are units (single digits)
- 3 x Three of the answers are units (single digits)
- 4 x Two of the answers are units (single digits)

2.7

Recording reasoning in sentences

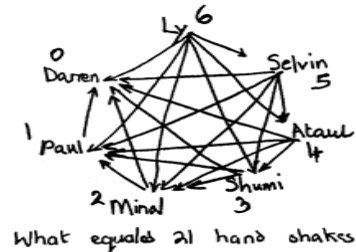
Table V is 1 times - I matched that quickly. For table Y $C \times C = AC$
 $6 \times 6 = 36$. This must be 6 times because $2 \times 3 = 9$ and $9 \times 9 = 81$. Then
 I looked at how many 2 digit numbers there were and found that
 it matched with 9 times table so the other one was 3 times table.

Table V is 1 times - I matched that quickly. For table Y, $C \times C = AC$,
 $6 \times 6 = 36$. This must be 6 times because $3 \times 3 = 9$ and $9 \times 9 = 81$.
 Then I looked at how many 2-digit numbers there were and found
 that it matched with 9 times table so the other one was 3 times table.

2.8

Recording in an annotated diagram

Each of 6 friends shakes hands once with each other person. How many handshakes are there?



2.9

Recording in a table

Each of 6 friends shakes hands once with each other person. How many handshakes are there?

Now I have proved the number of handshakes for each number of children I have drawn a chart to show how I got my answers and to help understand the rule.

N ^o of children	N ^o of handshakes	how I got my answer
2	1	+1
3	3	2+1
4	6	3+2+1
5	10	4+3+2+1
6	15	5+4+3+2+1
7	21	6+5+4+3+2+1

2.10

Recording reasoning with a calculation

Three rabbits had a total weight of 17 kg.
 The second rabbit weighed 2 kg more than the first rabbit.
 The third rabbit weighed 1.2 kg more than the first rabbit.

What is the weight of each rabbit?

$$\begin{array}{r}
 17-00 \\
 - 2-00 \\
 \hline
 15-00 \\
 - 1-20 \\
 \hline
 13-80 \div 3 = 4-60 \\
 \text{Rabbit 1 } 4-60 \quad \text{Rabbit 2 } 6-60 \quad \text{Rabbit 3 } 5-80
 \end{array}$$

2.11

Summary

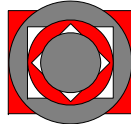


- Students will not learn how to conjecture and reason unless they have planned opportunities to do so
- Teachers can help students to reason by:
 - highlighting relevant vocabulary
 - questioning students in ways that focus their attention and stimulate their thinking
 - asking students to present their reasoning to different audiences
 - showing students examples of well-explained, well-presented reasoning

2.12

Mathematics workshop 4

Session 3: Data handling 1



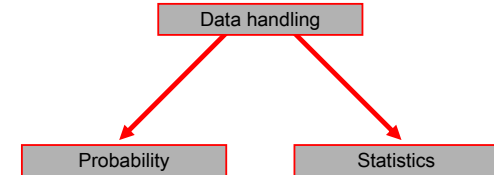
Objectives

By the end of this session you will:

- be familiar with the standards for data handling from Grade 1 to Grade 6
- understand the key elements of data handling
- be familiar with a range of problem-solving activities involving, collecting, organising, representing and interpreting data, and drawing conclusions

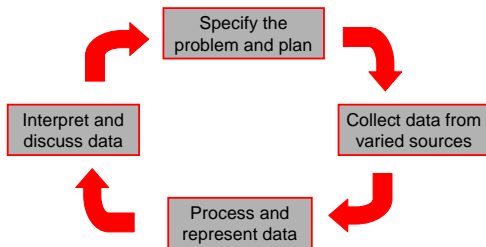
3.1

Planning, teaching and assessment



3.2

The data handling cycle



3.3

Making decisions

Questions for students to ask:

- What problem are we trying to solve?
- What information is needed?
- How do we collect the information?
- What do we do with the information?
- How do we represent it?
- What conclusions can we draw?
- Do we need to collect any more information?

3.4

Planning an enquiry

- What data should we collect?
- Will the data consist of measurements or opinions?
- How should we collect the data – by counting or measuring?
- If we collect data by counting, should we take a vote, or design a questionnaire?
- Whose opinion should we ask? Should everyone with an interest be asked or just a sample?
- How do we ensure that our samples are representative?

3.5

Planning an enquiry



- Should different groups have a weighting (e.g. should teachers' views carry more weight than parents' views)? Or should all opinions have the same value?
- How should results be analysed?
- How should results be presented? To whom? Does this influence how we should collect the data?
- What else needs to be considered?

3.6

Essential features of a graph



- **A title**
essential to knowing what the graph is about
- **Labels on the axes**
show what the data are and the units used
- **Numbering of the scale(s)**
shows how to read the values of the data
- **A key for two or more sets of data**
shows which line or bar refers to which data set
- **The source**
acknowledges the source of the data, where appropriate

3.7

Representing information



Tables

Tally chart
Frequency table
Lists
Two-way tables

Graphs and charts

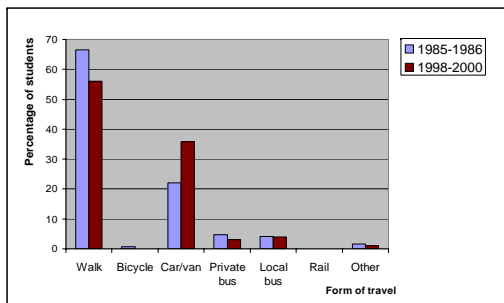
Block graph
Pictogram
Frequency bar chart
Frequency line graph
Pie chart

Logic diagrams

Carroll diagram
Venn diagram

3.8

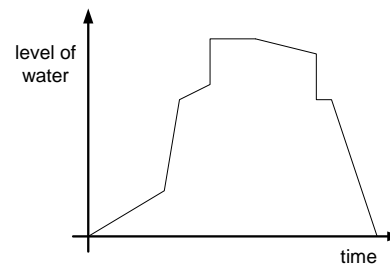
Main form of travel to school: students aged 5 to 11



Source: National Travel Survey, Department for Transport, UK

3.9

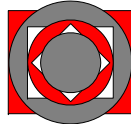
What 'story' goes with this graph?



3.10

Mathematics workshop 4

Session 4: Data handling 2



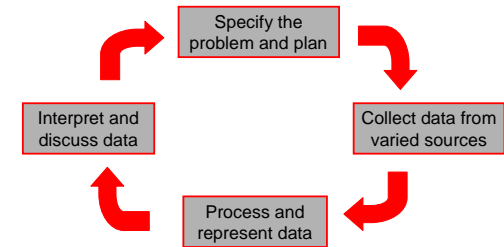
Objectives

By the end of this session you will:

- have discussed ways of using ICT in data handling
- have considered strategies for assessing students' knowledge and understanding of data handling

4.1

The data handling cycle



4.2

Data handling with a purpose

Is everyone happy at break times?

- Are there any problems?
- How could we find out?
- What could we do about it?
- How else could we improve break times?
- Could they be more fun or interesting?

4.3

Assessment

- Day-to-day assessment helps teachers to plan work at the right level
- It is important to:
 - find out whether students have been taught work in the earlier grades
 - assess whether students are secure with the key standards from earlier grades

4.4

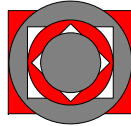
Summary

- Data handling activities should be purposeful and based on solutions to problems
- The best activities:
 - relate to students' experiences
 - involve each of the four stages of data handling
- There is a range of ways to explore data handling, including using ICT to collect, process and represent data
- Data handling skills have to be taught – students don't acquire them by accident
- Regular assessment ensures that students are ready for the next stage of the work

4.5

Mathematics workshop 4

Session 5: Division 1



Objectives

By the end of this session you will:

- have reviewed progression in division up to the end of Grade 3
- know how to use models, images and language in the teaching of division
- know how students can be helped to learn division facts

5.1

Task 1

Are there different 'stories' that can be represented by this expression?

$$12 \div 2$$

Think of three different stories to reveal different interpretations of division.

5.2

Possible interpretations of $12 \div 2$

- Equal sharing of 12 between 2
- Finding one half of 12
- Grouping 12 into 2s, which includes:
 - counting forwards in 2s from 0 to 12, or repeatedly adding 2s to reach 12
 - counting back in 2s from 12 to 0, or repeatedly subtracting 2 from 12

5.3

Task 2

What different ways are there of reading this calculation?

$$12 \div 2$$

5.4

The language of division 1

$12 \div 2$	$6 \div 12$
12 divided by 2	6 divided by 12
12 divided into 2	6 divided into 12
12 divided between 2	6 divided between 12

5.5

The language of division 2



- The use of '**divided by**' teaches students that, as with addition, subtraction and multiplication:
 - the structure and language of calculation follow a consistent pattern
 - the structure and language have an associated image that supports the method of calculation

5.6

Task 3



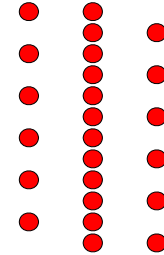
What images might help to secure a child's understanding of this calculation?

$$12 \div 2$$

5.7

Models of division for $12 \div 2$

Sharing between 2, or finding half of



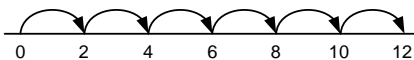
5.8

Models of division for $12 \div 2$

Grouping into 2s



Counting forwards (or backwards) in 2s



5.9

Models of division for $12 \div 2$

Repeatedly subtracting 2 from 12

$$\begin{array}{ll} 12 - 2 = 10 & 6 - 2 = 4 \\ 10 - 2 = 8 & 4 - 2 = 2 \\ 8 - 2 = 6 & 2 - 2 = 0 \end{array}$$

Deriving from knowledge of multiplication facts

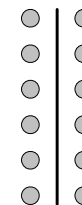
$$\begin{array}{ll} 6 \times 2 = 12 & \longrightarrow 12 \div 2 = 6 \\ 2 \times 6 = 12 & \longrightarrow 12 \div 6 = 2 \end{array}$$

5.10

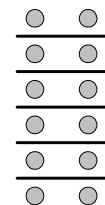
Models of division for $12 \div 2$

Sharing, or finding half

How many in one column?



How many rows are there?



5.11

Sharing:



- secures understanding of halving and one-to-one correspondence between objects
- requires little knowledge or skill beyond counting
- provides no image to support understanding of what to do with remainders
- as the divisor increases:
 - becomes more difficult to visualise
 - becomes less efficient

5.12

Grouping:



- secures understanding that the divisor is important in the calculation
- links to counting in equal steps on a number line
- requires knowledge of addition and subtraction facts
- provides an image to support understanding of what to do with remainders
- is more efficient as the divisor increases
- provides a firm basis on which to build students' understanding of division

5.13

Task 4



- What knowledge and understanding of division are students expected to have by the end of Grade 2?
- What is the expected progression across Grade 3?

5.14

Task 5



We expect students to know division facts corresponding to:

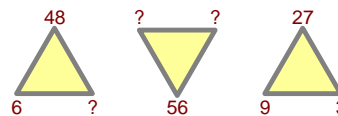
- Grade 2: tables for 2, 5 and 10, then 3 and 4
- Grade 3: all multiplication facts to 10×10
- Grade 4: all multiplication and division facts to 10×10

How can we help students to recall division facts?

5.15

Supporting the learning of division facts

Using 'tables trios'

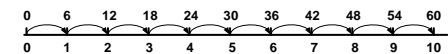


- I am thinking of a 'tables trio'. Two of the numbers are 48 and 6. What is the third number?
- One of the numbers is 56. What could the other two numbers be?
- What are the four facts associated with this trio?

5.16

Supporting the learning of division facts

Using a number line



Using a horizontal counting stick



5.17

Supporting the learning of division facts

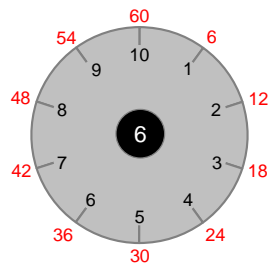
Using a vertical counting stick

0	0	$0 \div 6 = 0$
6	1	$6 \div 6 = 1$
12	2	$12 \div 6 = 2$
18	3	$18 \div 6 = 3$
24	4	$24 \div 6 = 4$
30	5	$30 \div 6 = 5$
36	6	$36 \div 6 = 6$
42	7	$42 \div 6 = 7$
48	8	$48 \div 6 = 8$
54	9	$54 \div 6 = 9$
60	10	$60 \div 6 = 10$

5.18

Supporting the learning of division facts

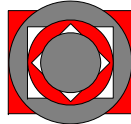
Using a calculation dial



5.19

Mathematics workshop 4

Session 6: Division 2



Objectives

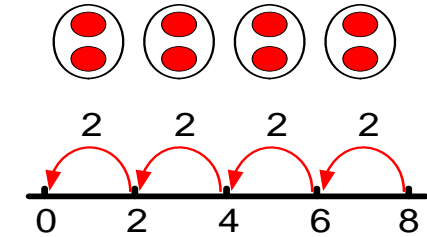


By the end of this session you will:

- have reviewed progression in division in Grades 4, 5 and 6
- know how a number line can be used to teach division, including representing the quotient as a fraction
- have considered approaches to mental and written division calculations

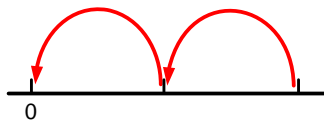
6.1

Number lines and grouping



6.2

Task 1

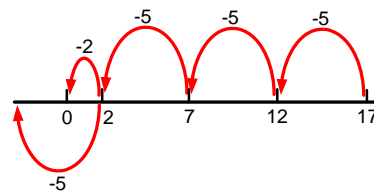


What division calculation is represented if:

- the step size is 4?
- the right-hand marker represents 18?
- the middle marker represents 6?

6.3

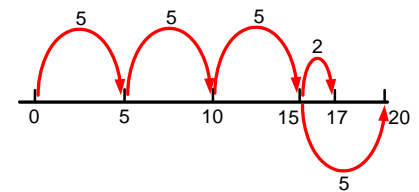
Remainders



$$17 \div 5 = 3\frac{2}{5}$$

6.4

Remainders



$$17 \div 5 = 3\frac{2}{5}$$

6.5

Task 2



Does this calculation have different answers in different contexts?

$$22 \div 4$$

When should a remainder be expressed as a whole number?

When should the quotient be expressed as a fraction or a decimal?

6.6

Informal recording of 43×6

$$\begin{array}{r}
 43 \\
 40 + 3 \\
 \downarrow \quad \downarrow \\
 240 + 18 = 258
 \end{array}$$

Answer: 258

6.7

Informal recording of 43×6

x	40	3	$240 + 18 = 258$
6	240	18	

Answer: 258

6.8

Informal recording of $84 \div 7$

$$\begin{array}{r}
 84 \\
 80 + 4 \\
 70 + 14 \\
 \downarrow \quad \downarrow \quad \div 7 \\
 10 + 2 = 12
 \end{array}$$

Answer: 12

6.9

Informal recording of $84 \div 7$

x	10	2	$10 + 2 = 12$
7	70	14	

Answer: 12

6.10

Progression from the grid method ...

27×34
Approximate answer: $30 \times 30 = 900$

x	20	7	810 ← This is 27×30
30	600	210	
4	80	28	108 ← This is 27×4
			918 ← This is 27×34

Answer: 918

6.11

... to an efficient standard method

27×34
Approximate answer: $30 \times 30 = 900$

$$\begin{array}{r} 27 \\ \times 34 \\ \hline 810 \\ 108 \\ \hline 918 \end{array}$$

← This is 27×30
← This is 27×4
← This is 27×34

Answer: 918

6.12

Progression from 'chunking' ...

$560 \div 24$

Approximate answer:
 $550 \div 25 = 22$

$$\begin{array}{r} 560 \\ 24 \times 10 = 240 \quad - \underline{240} \\ \hline 320 \\ 24 \times 10 = 240 \quad - \underline{240} \\ \hline 80 \\ 24 \times 2 = 48 \quad - \underline{48} \\ \hline 32 \\ 24 \times 1 = 24 \quad - \underline{24} \\ \hline 8 \end{array}$$

Answer: 23 R 8

6.13

... to efficient 'chunking' ...

$560 \div 24$

Approximate answer:
 $550 \div 25 = 22$

Answer: 23 R 8

$$\begin{array}{r} 560 \\ 24 \times 20 \quad - \underline{480} \\ \hline 80 \\ 24 \times 3 \quad - \underline{72} \\ \hline 8 \end{array}$$

6.14

... to an efficient standard method

$560 \div 24$

Approximate answer:
 $550 \div 25 = 22$

$$\begin{array}{r} 24 \overline{) 560} \\ \underline{20} \\ 480 \\ \underline{80} \\ 3 \\ \underline{23} \\ 8 \end{array}$$

Answer: 23 R 8

6.15

Summary



Grades 3, 4 and 5 should focus on:

- how to use both factorising and partitioning as mental strategies for division
- how to record these strategies to support or explain their thinking
- how and when to express a quotient with a remainder, or as a fraction or decimal (a model of a number line is helpful here)

6.16

Summary



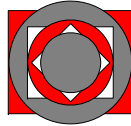
Grades 5 and 6 should focus on:

- making informal written methods for division, such as 'chunking', as efficient as possible, as in the long division method
- preparing confident students to carry out 'short' division of a three- or four-digit number by a single-digit number

6.17

Mathematics workshop 4

Session 7: Division 3



Objectives



By the end of this session you will:

- have discussed approaches to teaching short division
- have considered some lessons and resources to support the teaching of division

7.1

To divide 81 by 3

$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

$$\begin{array}{r} 27 \\ 3 \overline{) 81} \end{array}$$

7.2

To divide 672 by 4: 'hundreds' stage

$$\begin{aligned} 672 \div 4 &= (400 + 272) \div 4 \\ &= (400 \div 4) + (272 \div 4) \\ &= 100 + (272 \div 4) \end{aligned}$$

$$\begin{array}{r} 1 \\ 4 \overline{) 672} \end{array}$$

7.3

To divide 672 by 4: 'tens' stage

$$\begin{aligned} 272 \div 4 &= (240 + 32) \div 4 \\ &= (240 \div 4) + (32 \div 4) \\ &= 60 + (32 \div 4) \end{aligned}$$

$$\begin{array}{r} 16 \\ 4 \overline{) 672} \end{array}$$

7.4

To divide 672 by 4: 'units' stage

$$\begin{aligned} 272 \div 4 &= (240 + 32) \div 4 \\ &= (240 \div 4) + (32 \div 4) \\ &= 60 + (32 \div 4) \\ 32 \div 4 &= 8 \end{aligned}$$

$$\begin{array}{r} 168 \\ 4 \overline{) 672} \end{array}$$

7.5

Types of short division calculations

no exchange, no remainder	4)848	
no exchange, with remainder	3)635	
with exchange, no remainder	7)994	
with exchange, with remainder	3)470	
empty place at start of quotient	7)287	
noughts in the quotient	4)816	8)5608
decimal dividend	5)61.5	3)4.26

7.6

Six lessons on division

- Doubling and halving
- Multiplication and division facts and $TU \times U$
- Multiplication and division as inverse operations
- $HTU \div U$ (whole-number answers)
- Expressing a quotient as a fraction or decimal
- Tests of divisibility

7.7

Task 1: Six lessons on division

Browse through the six lessons.

Consider which you might choose to teach.

Make some preliminary decisions about the possible modifications that you might make to suit your particular students.

7.8

Recommendations

Put more emphasis on:

- developing from informal methods to formal approaches
- representing remainders as fractional or decimal parts
- dividing by 10, 100 and 1000
- using inverse operations rather than trial and error in 'missing number' and 'what is my number?' problems, e.g. $527 \div \square = 31$
- working with ratios, e.g. dividing a line in a given ratio
- solving problems involving proportion, e.g. finding the cost of 10 cakes given the cost of 6 cakes

7.9

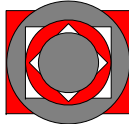
Following up the day on division

- Read, reflect on and annotate the article *Continuing division*
- Think carefully about the points in the summary
- Consider whether you need to adjust your teaching plans in any way – if so, make notes on what needs to be done and discuss these with your colleagues

7.10

Mathematics workshop 4

Session 8: Early years



Objectives



By the end of this session you will:

- understand the Kindergarten standards for mathematics
- be aware of some features of effective teaching and learning in Kindergarten and Grade 1

8.1

Kindergarten standards



- Say and use the number names in order in familiar contexts
- Count reliably up to 10 everyday objects
- Understand 'zero'
- Recognise numerals 0 to 10
- Use language such as 'more' or 'less', 'greater' or 'smaller', 'heavier' or 'lighter', to compare two numbers or quantities

8.2

Kindergarten standards



- Find one more or one less than a number from 1 to 10
- Begin to relate addition to combining two groups of objects, and subtraction to 'taking away'
- In practical activities and discussion, solve simple practical problems involving adding or subtracting, explaining methods orally

8.3

Kindergarten standards



- Use language such as 'circle' or 'bigger' to describe the shape and size of solids and flat shapes
- Use everyday words to describe position, or the time of day
- Talk about, recognise and re-create simple patterns
- Classify objects using own criteria
- use developing mathematical ideas to solve practical problems

8.4

Some children will be able to:



- say, read and order numbers from 0–20
- write familiar numerals
- compare more than two quantities
- remember double of small numbers

8.5

Social skills



- Listening in small and large groups
- Showing and talking about, in front of other children, what they have been doing
- Responding to direct questions
- Finding, using and putting away equipment
- Taking turns
- Doing something with a partner
- Playing a game and following 'rules', e.g. the repetitive structures of throwing dice and collecting objects

8.6

Assistants can:



- sit near children who need help and do this quietly and discreetly
- help a group to work together sociably
- help them to understand how to do an activity
- ask children questions to get them thinking
- introduce or reinforce mathematical vocabulary
- help a particular group feed back to the whole group
- observe particular children and feed back to the teacher on their responses

8.7

A Grade 1 'lesson'

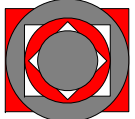


- An introduction with the whole class which will almost always include some counting
- Some teaching of the whole class on the main mathematics topic of the day
- Group activities, with extra adult support if available
 - for everyone in small groups simultaneously
 - for one or more play activities, linked to the theme of the lesson, worked on by groups in turn during the day
- Discussion time with the whole class after the group activities are ended

8.8

Mathematics workshop 4

Session 9: Geometry and measures 1



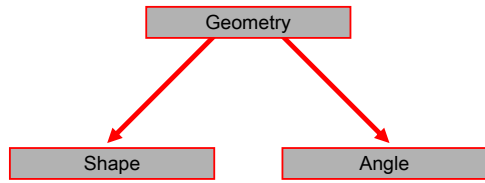
Objectives

By the end of this session you will:

- be familiar with the standards for geometry from Grade 1 to Grade 6
- understand the key elements of transformations
- be familiar with a range of problem-solving activities involving shape, space and constructions

9.1

Geometry

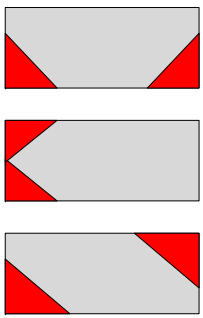


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    graph TD
      Geometry[Geometry] --> Shape[Shape]
      Geometry --> Angle[Angle]
  
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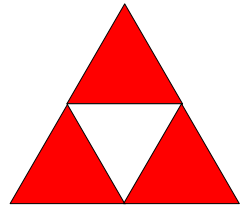
9.2

Visualising 1



9.3

Visualising 2



9.4

Visualising - tetrahedron

- How many edges are there?
- How many edges separate two faces of different colours?
- How many edges separate two faces of the same colour?
- What are the colours of the faces that meet at each vertex?
- Imagine placing the shape on a surface.
What colours can be seen?
What colour is hidden?

9.5

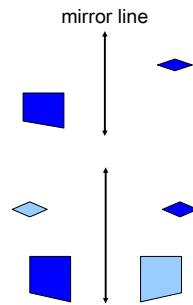
Reflection



- When an object is reflected in a mirror, the image is:
 - congruent to the original (the same size and shape)
 - reversed
 - the same distance behind the mirror as the object is in front
- The mirror is usually represented by a straight line (the mirror line)
- The mirror line is double-sided, reflecting objects both ways

9.6

Reflection



9.7

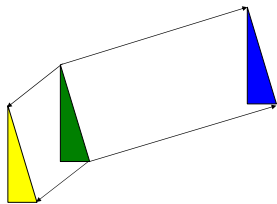
Rotation



- The shape and size of the object remain constant during the rotation and the image is congruent to the original
- Every rotation has associated with it:
 - a centre of rotation (the point around which the object turns)
 - the angle of rotation (the angle through which it turns)
 - the direction of rotation (whether the rotation is clockwise or anticlockwise)

9.8

Translation



9.9

Translation

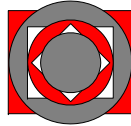


- When an object is translated, the image is congruent to the original (the same shape and size)
- The orientation stays the same

9.10

Mathematics workshop 4

Session 10: Geometry and measures 2



Objectives

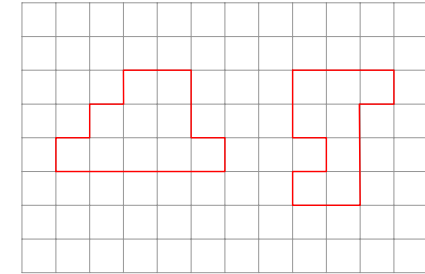


By the end of this session you will:

- be familiar with the standards for geometry from Grade 1 to Grade 6
- understand the progression in work on area
- have experienced a range of problem-solving activities involving constructions, perimeter, area and volume

10.1

Perimeter and area



10.2

Area – progression in standards



- Grade 3: understanding what parts of a shape are being measured and how to measure them
- through to:
- Grade 5: linking 3-D shapes and measures through finding a volume
 - Grade 6: using formulae to find the areas of triangles and parallelograms

10.3

Area – progression in activities



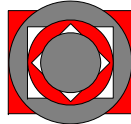
Solving problems by:

- direct comparison: placing one shape on top of another
- using non-standard units: covering the shape with counters or coins and counting
- using standard units: using square tiles or squared paper
- using formulae: finding areas of rectangles using the lengths of the short and long sides

10.4

Mathematics workshop 4

Session 11: Division and assessment



Objectives



By the end of this session you will:

- have analysed test questions on division as an aid to assessment
- have considered the errors that students may make with division and the implications for teaching

11.1

Task 1



What do students need to know and be able to do before they move on to division calculations that extend beyond 'tables facts'?

11.2

Division: Grades 2 and 3



Written

- interpretation of words such as:
half, pair, share, left over, divided exactly by
- recognition of multiples of 10
- simple word problems involving grouping of two-digit numbers into 2s, 4s, 5s, 6s or 10s
 - sometimes with a remainder
 - sometimes involving rounding up

11.3

Division: Grades 3 and 4



Mental

- division facts

Written

- identifying multiples, e.g. of 3 and 5
- word problems involving:
 - TU or HTU \div U (or multiple of 10), sometimes with remainder, sometimes involving rounding
 - mixed units (e.g. 4 metres \div 50 centimetres)
- missing-number questions involving inverses, and 'I am thinking of a number' problems

11.4

Division: Grades 4 and 5



Mental

- division of TU or HTU by U

Written and calculator

- short division (e.g. $847 \div 7$)
- word problems involving HTU by U or TU and:
 - rounding of a decimal answer
 - harder mixed units (e.g. $12.3 \text{ kg} \div 300 \text{ g}$)
 - simple direct proportion
- more complex missing-number and 'I am thinking of a number' problems

11.5

Division: Grades 5 and 6



Mental

- division of decimals by 10
- simple direct proportion

Written and calculator

- short division
- word problems involving more interpretation or steps:
 - rounding of decimal answers
 - mixed units (e.g. 10 m ÷ 9.2 cm, 3 kg ÷ 60 g)
 - direct proportion
- missing-number and 'I am thinking of a number' problems involving decimals

11.6

Criteria for calculation methods



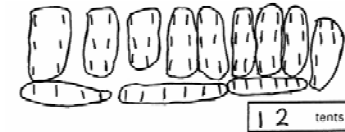
A calculation method for division should be:

- reliable (the student gets the right answer)
- appropriate (it is suitable for the type of calculation and the available tools: mental, written, calculator)
- efficient (it is not too time-consuming)
- checkable

11.7

An inefficient and inappropriate method

A tent holds 6 boys.
How many tents are needed to hold 70 boys?



Grade 5 student

11.8

An inefficient and inappropriate method

$568.1 \div \square = 24.7$
(calculator allowed)

$$568.1 + \boxed{23} = 24.7$$

Grade 5 student, who divided by 5, 6, 8, 16 and 20 before trying 23.

11.9

Task 5: Students' errors



- Consider each error. Is it caused by:
 - a careless slip (e.g. a division fact recalled incorrectly)?
 - basic misunderstanding of place value?
 - incorrect or inappropriate application of a method?
 - another reason?
- Annotate each example with your analysis.

11.10

Task 5: Students' errors



- Take one of the errors. What could you do about it as a teacher:
 - to avoid the error happening in the first place?
 - to model and explain the correct approach?
- If you have time, repeat with another error.

11.11

Summary



- Don't merely re-teach the method
- Analyse each error and its cause
- Point out careless slips – get the student to learn facts 'by heart'
- Tackle misunderstanding of place value by the use of place value cards, Gattegno charts, multibase blocks, calculators, ... Stress that when multiplying and dividing by a power of 10 digits move to the left or right
- Model remainders on a number line

11.12

Summary

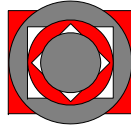


- Misapplication of a division method may be caused by:
 - lack of understanding of partitioning and the principle of the distributive law of division – more work on informal recording of mental methods of division is needed
 - being moved on too quickly, without a thorough grasp of all the prerequisite skills

11.13

Mathematics workshop 4

Session 12: Direct proportion



Objective

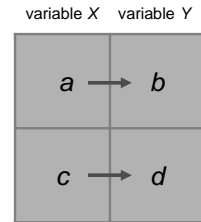


By the end of this session you will:

- understand direct proportion and know how to teach it

12.1

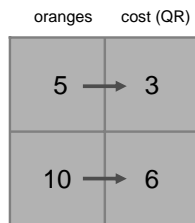
Structure of a direct proportion problem



The ratio of variable X to variable Y is always the same, so a is to b as c is to d . We say that the two variables are in 'direct proportion'.

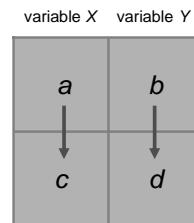
12.2

Structure of a direct proportion problem



12.3

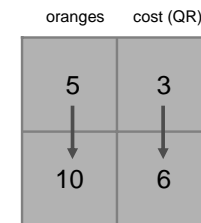
Structure of a direct proportion problem



It is also true that a is to c as b is to d .

12.4

Structure of a direct proportion problem



12.5

Solving direct proportion problems



- When you solve problems involving direct proportion in this way, you can work either left to right (across the variables) or top to bottom (within the variables)
- The most efficient way will depend on which numbers are known and the relationships between them

12.6

A recipe for 6 people needs 12 eggs.
Adapt it for 8 people.

people	eggs
6	→ 12
8	→ 16

→
x 2

Answer: 16 eggs

12.7

A recipe for 6 people needs 4 eggs.
Adapt it for 9 people.

people	eggs
6	4
↓ 9	↓ 6

↓ x 1.5

Answer: 6 eggs

12.8

A recipe for 6 people needs 120g flour.
Adapt it for 7 people.

people	flour (g)
6	→ 120
7	→ 140

→
x 20

Answer: 140g flour

12.9

A recipe for 8 people needs 500g flour.
Adapt it for 6 people.

people	flour (g)
8	500
↓ 6	↓ 375

↓ x $\frac{3}{4}$

Answer: 375g flour

12.10

A recipe for 6 people needs 140g flour.
Adapt it for 14 people.

people	flour (g)
6	140
14	?

This time there is no obvious straightforward relationship.

12.11

A recipe for 6 people needs 140 g flour.
Adapt it for 14 people.

people	flour (g)
6	140
1	23.333
14	326.666

Answer: approximately 327 g

12.12

Informal working: cost of 4.5 kg bananas

weight (kg)	cost (QR)
1	3.50
2	7.00
4	14.00
0.5	1.75
4.5	15.75

Answer: QR 15.75

12.13

Efficient working: cost of 4.5 kg bananas

bananas (kg)	cost (dirhams)
1	350
4.5	1575

↓ × 4.5

Answer: QR 15.75

12.14

Efficient working: cost of 4.5 kg bananas

bananas (kg)	cost (dirhams)
1	350
4.5	1575

→ × 350

Answer: QR 15.75

12.15

Mass of tomatoes that QR 2.75 will buy

cost (QR)	tomatoes (kg)
14.25	4
1	$4 \div 14.25$
2.75	0.772

Answer: 0.772 kg or 772 g

12.16

Solving direct proportion problems



- To get started with a direct proportion problem, put the information in a four-cell diagram, making sure that the units are consistent for each variable
- Arrange the diagram so that the unknown is in the bottom right corner (but this is not essential)
- Look for a relationship that attracts you – across (left to right) or within the variables (top to bottom)
- Apply the same relationship to find the unknown number

12.17

Solving direct proportion problems

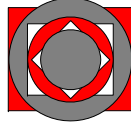


- Students will move from informal methods using a four-cell diagram in Grades 5 and 6 to the generally applicable unitary method in later grades
- Some primary students may use the unitary method without being taught it, developing it for themselves

12.18

Mathematics workshop 4

Session 13: Numeracy and ICT



Objective



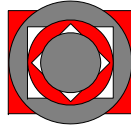
By the end of this session you will:

- be familiar with some small programs that support the development of number skills in the primary grades

13.1

Mathematics workshop 4

Session 14: Using a calculator



Objectives



By the end of this session you will:

- have considered how a calculator can support students' understanding in mathematics
- be familiar with the use of a calculator in the mathematics standards
- be aware of a range of problem-solving activities involving the use of a calculator

14.1

The mathematics curriculum



- Learning mathematics involves developing ideas and skills, and consolidating and practising them
- Using mathematics involves applying the ideas and skills that have been learned in everyday experiences and in mathematical problem solving

14.2

Reinforcing number concepts



- Use any of these keys as often as you like:

[1] [0] [5] [+] [=]

Make each of these numbers. Press as few keys as possible.

16 37 88 638

- One whole number is divided by another on a calculator. Each of the whole numbers is less than 20. The display shows 0.5384615. What are the numbers?

14.3

Types of problems



- Everyday problem solving
- Mathematical problem solving

14.4

Everyday problem solving



Three important processes:

- deciding what calculations to do
- carrying out the calculations
- making sense of the answers in the context of the problem

Three roles for the calculator:

- giving the correct answer
- allowing attention to remain on the problem
- giving confidence to try out specific cases

14.5



Three important processes:

- trying something out
- predicting a pattern
- explaining a pattern

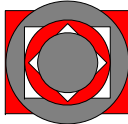
Three roles for the calculator:

- giving the correct answer
- allowing attention to stay on the problem and mathematics
- making it easy to try out specific cases

14.6

Mathematics workshop 4

Session 15: The interactive whiteboard



Objectives



By the end of this session you will:

- have viewed and discussed some activities using the interactive whiteboard
- have discussed the benefits of interactive teaching programs and specialist software

15.1

Uses of ICT in mathematics



- basic, scientific, statistical and graphics calculators
- function graph plotters, e.g. *Autograph*
- programming languages, e.g. *Logo*
- dynamic geometry systems, e.g. *Geometer's sketchpad*
- spreadsheets, databases, e.g. Excel
- presentations, e.g. PowerPoint
- small programs
- the Internet
- interactive whiteboards

15.2

New technologies:

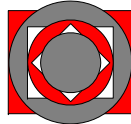


- motivate students and teachers
- engage students and hold their attention
- provide a multimedia approach to teaching and learning
- provide powerful visual images
- improve teacher efficiency

15.3

Mathematics workshop 4

Session 16: Summing up



Objectives



By the end of this session you will:

- have watched videos tailored to the local context
- have reflected on the workshop

16.1

Aims of the workshops



To help subject leaders and teachers to:

- get to know the new mathematics standards
- consider the implications of the standards for planning, teaching and assessment
- begin planning or refine a mathematics scheme of work
- support colleagues as they implement the standards

16.2

Review: planning



- Are you working together as a team to plan your teaching and assessment within your school?
- Do you exchange lesson plans with each other?
- How well developed is your scheme of work? Do colleagues use it to help them to plan lessons? Or do they plan lessons by looking at the next chapter of the textbooks?
- To what extent are problem solving and the use of ICT integrated into your scheme of work and lesson plans?

16.3

Review: teaching



- Do you have a better understanding of what students should be taught in the grades that you teach?
- Do you have some new ideas to help you to teach your students?
- Do you understand the need to work closely with your colleagues to ensure that you are using consistent teaching methods?
- Can you share ideas across schools?

16.4

Review: assessment



- Do you understand that assessment does not equate to 'multi-choice tests'?
- Do you know why diagnostic assessment is an important feature of planning lessons?
- Do you have some new ideas for assessment activities?

16.5

Review: resources



- Do you have all the apparatus and equipment that you need to support your teaching and students' learning? What about ICT?
- How will you make sure that there is progression for the students in your use of ICT?
- Do teachers have sufficient expertise to use ICT in their teaching?
- Can you share ideas for the use of resources across schools?

16.6