

Mathematics lessons for Grade 10

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Using these lesson plans

Each of these sample lessons for Grade 10 is suitable for use with a whole class. The lessons are standalone examples to illustrate different teaching and learning activities. They are not intended to be taught as a sequence.

The first two lessons are based on the Grade 10 standards for foundation mathematics. The third and fourth lessons are based on the Grade 10 standards for advanced mathematics. The relevant standards are shown in the lesson plans.

The lessons are drawn from different topics to show spread rather than sequence. They illustrate different pedagogical styles for the various topics and the different sets of standards.

Each lesson has been organised in three parts: a starter to introduce the lesson, a main activity, and a final phase to help students to reflect on the lesson and consolidate their learning. Early in the lesson, you should outline the purpose of the lesson, drawing out for students what they will learn and how this builds on previous work. In the final part of the lesson, you will need to establish the key learning points, what students should remember and what they will go on to learn next about the topic. You should also help students to appreciate the links that can be made to other topics in mathematics, in other subjects or in the real world.

The lesson plan suggests how you as the teacher might teach the class and interact with students but you will need to add your own questions to present and develop the mathematical threads that run through the lesson. The plan makes no attempt to tell students how or when to take notes of the lesson.

The lesson plans do not include homework tasks because the lessons are single examples taken out of sequence. You will need to provide homework since it is an important part of a lesson.

There may be too much material in the lesson plan for one lesson (e.g. of 60 minutes), since the rate of teaching and learning will depend on the class. In this case, you might designate one of the activities in the lesson as homework, or carry it forward to the next lesson. Be selective about which activity to cut – it does not have to be the last one merely because it comes at the end.

You may need to supplement the activities in the lesson with simpler or more challenging tasks if the students in your class have a range of attainment. You could choose from activities in textbooks or from your own resources. If you wish, different tasks can be given to different groups of students, according to their needs. Each lesson plan includes some suggestions for further tasks.

Answers to some questions are provided to help you to correct students' responses and give feedback. Many of the questions are open-ended and have more than one possible answer. They are there to facilitate critical discussion, to help students to generalise and to help them gain insight. They illustrate style and a pedagogical approach rather than the words you should use yourself.

The success of the lesson will depend on you and your class. You know your students' strengths and weaknesses better than anyone else. Each lesson plan is there to help you to think about how to conduct a lesson in its chosen topic area. It is not a prescription of what to do.

10.1

Foundation

Using visual images in number and algebra

Objectives

- Calculate with any real numbers, including mental calculations.
- Understand that the transformation of algebraic objects generalises the well-defined rules of arithmetic.
- Use brackets and correct order of precedence of operations when performing numerical or algebraic calculations.
- Multiply any combinations of monomial and binomial expressions, collecting and simplifying similar terms.
- Sum arithmetic sequences and give a ‘geometric proof’ for the formulae of these sums.

Starter

Inform the class that the lesson is to help develop facility in calculation and problem solving in number and algebra by using visual images.

Start with some quick response questions:

Q What is $9 \times (-3)$?

Q What is $4 - (-7)$?

Q What is $\sqrt{5^2 + 12^2}$?

Q Which of the following is a correct response to ‘find $\sqrt{a^2 + b^2}$ ’?

A $\sqrt{a^2} + \sqrt{b^2}$

B $a + c$

C It cannot be calculated or simplified without additional information.

Q What is the next number in this sequence: 1, 3, 6, 10, ...? What are these numbers called?

Q What is the sum of the first four positive integers?

Q Which of these are the same?

(a) 5.25×3.5 and $3^{1/2} \times 5^{1/4}$ (b) $6 + 7$ and $7 + 6$

(c) $8 - 4$ and $4 - 8$ (d) $8 - 4$ and $-4 + 8$

(e) $9 \div 6$ and $6 \div 9$ (f) $(-10) \div 2$ and $10 \div (-2)$

Q Which of the following pairs of statements are equivalent?

(a) $a \times b$ and $b \times a$ (b) $a - b$ and $b - a$ (c) $b \div a$ and $a \div b$

(d) $(a + b) \times c$ and $a \times c + b \times c$ (e) $c \div (a + b)$ and $c \div a + c \div b$

Q Is $50 + (30\% \text{ of } 50)$ the same as 1.3×50 ? Explain your answer using complete sentences.

Q Is $90 - (80\% \text{ of } 90)$ the same as 0.2×90 ? Explain your answer using complete sentences.

Main activity

Vocabulary

negative numbers
positive integers
brackets
order of precedence
percentage
commutative
distributive
triangular numbers
arithmetic series

Resources

OHTs 10.1a–g

Say that it is important to develop skills for thinking about problems from different perspectives. A useful maxim is:

Think numerically, think algebraically and think geometrically.

Tell students that the more flexible they are in their methods of approaching mathematical problems and calculations, the better they will be at doing them.

Show **OHT 10.1a**. Ask students what patterns they can spot in the table. Now ask what they think the table represents. Make sure that all agree it is part of a multiplication table.

Now get the students to say what happens when the table is extended to the *left*. Then ask:

Q What happens when this enlarged table is extended *downwards*?

Show **OHT 10.1b**. Point out that this table is what the students have just been discussing and that it shows the result of multiplying a number in the reference row by a number in the reference column, both of which are in bold fonts. Now ask the important question:

Q What do you notice that is special in this table?

Make sure that students observe that the product of one negative integer and one positive integer is a negative integer and that the product of two negative integers is a positive integer. Shade different regions of the table to emphasise these facts, and point out the symmetries in the shaded table.

Conclude this part of the lesson by pointing out the power of the visual image to understanding how to work with negative numbers.

Next, show students how to visualise a quantity increased or decreased by a certain percentage; **OHT 10.1c**. The important point to get across here is the decimal representation for the effect of the percentage increase or decrease, and how the mere act of thinking about what the visual image would like can lead to an economy of the mathematics used to calculate the effects of the percentage change.

The next two activities are purely algebraic. First demonstrate how areas of rectangles can be used to develop the rules of multiplication when working with symbols. Use **OHT 10.1d**.

Q The top figure in the slide could be rotated through 90° . Does its area change? Using ‘area = width \times height’ what would be the expression for the area now? What do you conclude from this?

Once the result

$$ab = ba \quad (1)$$

has been suggested, stress that this is a basic property of multiplication; it does not matter in what order two numbers are multiplied. Ask if anyone knows the word to describe this characteristic (we say that multiplication is *commutative*).

Q Can you think of an operation between two numbers that is not commutative?

Now concentrate attention on the middle part of the slide. Check that everyone understands how the diagrams lead to the result

$$ac + bc = (a + b)c \quad (2)$$

Q Does it matter whether the diagrams are viewed from left to right or from right to left? (no)

Say that this implies that equation (2) can be read from right to left instead of from left to right as written above. Stress that it is important to do this without thinking: an equation read from left to right is exactly the same if read from right to left, but sometimes one form is more directly useful than the other. One side of these equations represents a factorisation while the other side represents an expansion of the factors.

Now concentrate on the lower portion of the slide. Here an area is removed to give the mathematical result

$$(a - b)c = ac - bc \quad (3)$$

Observe that the removal of area here corresponds to the subtraction within the bracket.

Q Does anyone know the name for rules of type (2) or (3)? (multiplication is distributive over addition or subtraction)

Now challenge students by asking:

Q Can anyone suggest what results are obtained if c is replaced by the more complicated expression $(c + d)$ in (2) or (3)?

Develop the discussion if any student volunteers a correct response and show how the expressions can be simplified in a series of stages by repetition of the rules already established. If no student suggests the correct answer, some students could be set this challenge as extension work (see below).

Now use **OHTs 10.1e** and **10.1f** to show how the two important results

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a - b)^2 = a^2 - 2ab + b^2$$

can be visualised.

As a final example, an altogether different sort of visualisation is demonstrated. Remind the class that sequences of the form

$$\begin{aligned} &5, 8, 11, 14, \dots \\ &a, a + 2, (a + 2) + 2, \dots \\ &a, a + d, (a + d) + d, \dots \end{aligned}$$

are all examples of arithmetic progressions (or AP for short).

Q What is the key characteristic of an AP? (each term differs from the preceding term by the same amount, d in the last example; d is called the constant difference)

Explain to the class that you will now show them a useful picture (**OHT 10.1g**) to sum an arithmetic series (sum of the first n terms of an AP) in which the first term is a and the constant difference is d . Students should imagine that each 'stair' is a rectangular block of width 1 and successive heights $a, a + d, (a + d) + d$ etc. There are n stairs so the width of the 'staircase' is n .

Steer the class to answer these questions:

Q What does l represent? (the value of the last term)

Q How does each half of the picture show the sum of the first n terms of the AP? (the area of the staircase is the sum of the n terms)

Q What does the total area of the rectangle formed by locking the two halves together represent? (twice the required sum)

Q What is the total area of the rectangle? (area is $n(a + l)$)

Q What is the sum of the first n terms of the AP? ($n(a + l)/2$)

Further work

- 1 Given equations (2) and (3), explore the effect of replacing c by $(c + d)$ or by $(c - d)$.
- 2 Specify a product of a monomial and a binomial expressions with linear terms and find a geometric representation for the product. Do this for several different cases. Generalise to a representation for a product of two binomial expressions with linear terms.
- 3 Find a geometric representation for $a^2 - b^2$.
- 4 Find a geometric representation for the sum of the first n positive integers that differs from the staircase representation of OHT 10.1g. Solve problems involving arithmetic series.

Consolidation

Ask students to summarise briefly what they have learned from the lesson.

Summary for students

- Visualisation provides insights into arithmetic and algebraic processes.
- Tabulation often reveals inherent patterns in structures.
- Area is used to represent multiplication of two numbers.

10.2

Foundation

Objectives

Using trigonometry

- Calculate the interior and exterior angles of regular polygons.
- Know the standard trigonometric ratios; use these ratios to find the angles of a right-angled triangle given two sides, or to find the remaining side given one side and one angle.
- Use Pythagoras' theorem to solve right-angled triangles.

Starter

This lesson uses the geometry and trigonometry of right-angled triangles to solve problems.

As a quick mental refresher, check that students can define sine, cosine and tangent in terms of the ratio of sides of a right-angled triangle and that they can state Pythagoras' theorem correctly. Check also that they know how to calculate the interior angle of a regular polygon.

Remind students how to use the sin, cos and tan function keys on their calculators and how to use the inverse functions with the same keys.

Main activity

Vocabulary

angle of elevation
angle of depression
altitude
clinometer

Resources

Stiff card, protractor,
heavy bob on the end
of a string, drawing
pins or any other form
of a simple axis of
rotation etc.
Very long measuring
tape
Resource 10.2a, one per
student or pair of
students

This lesson will be about practical trigonometry. (It will need one or two sessions.)

Challenge students with questions such as:

- Q How do surveyors know the height of Mount Everest?**
- Q How do we calculate how far a ship is out at sea from a land sighting?**
- Q How can the average speed of an ascending balloon be calculated by observing its position at two different times?**

The answer to all these questions is to use trigonometry!

It is useful to stress that on the Earth's surface trigonometric measurements of angle are made with reference to the horizontal or vertical directions. Discuss very briefly the meanings of *angle of elevation* (the angle looking up at a point from the horizontal) and *angle of depression* (the angle looking down to a point from the horizontal). Explain that when the elevation refers to the Sun or a star it is called the *altitude*.

Split the students into two groups and allow them a short time to invent and make a simple device to measure angles of elevation or depression. Give them access to suitable materials to do this. Such a device is called a *clinometer*.

If you prefer, you could prepare two clinometers in advance. This would save time for the main activities of the lesson.

Making and using a clinometer

- 1 To make a crude instrument, join two rods, or stiff pieces of card, so that one arm can rotate relative to the other about a swivel point. Angles are measured by keeping one arm horizontal and moving the other arm so that it aligns with the line of sight to a point on a distant object. The device must be fairly rigid for it to be moved carefully so that the angles between the two arms can be measured with a protractor.

- 2 Attach the base of a semicircular protractor to a rod or piece of stiff card and suspend a small heavy bob on a short string from the centre of the protractor. The protractor is then part of the clinometer and measurements will be more accurate. Make a sighting to a point on a distant object by looking along the rod. The suspended bob will always hang vertically and will make an angle with the 90° line on the protractor which gives the angle of elevation or depression.

You could ask students to explain the angle relationships in the type (2) model.

One group should now be sent outside with both clinometers and the long measuring tape. If no measuring tape is available, students will need to use a sensible method to estimate distance (number of leg strides, for example) in terms of some length that can be measured. The activity for this group is to estimate (as accurately as possible) the height of the highest point on the school building and the distance away of an inaccessible object such as a palm tree, flag-pole or another building. Group members should discuss among themselves how best to do these two tasks: who will take the measurements, who will record the measurements, how they will do the calculations, and so on. They can come back into class to ask for advice. They should produce good estimates of the two quantities required, and write up a clear, concise account of what they did and how they calculated results.

The other group should remain behind and work through the questions on **Resource 10.2a**. Go round, helping students individually if they are stuck. Try not to reveal the answer in helping a student with difficulties; instead ask more open-ended questions that will give them a clue as to how to continue. Check their solutions and give suggestions as to how they could attempt to correct mistakes.

In the second session, reverse the activities of the two groups. End by getting the groups to compare the methods and findings from their practical activity.

Further tasks

It is relatively straightforward to set harder tasks and questions on trigonometry. Many textbooks have good examples to choose from.

Able students could be directed to Grade 11 work to solve problems with triangles other than right-angled triangles, or harder problems in three dimensions that require repeated application of the basic principles of trigonometry.

Consolidation

Ask students to summarise what they have learned. In particular, the different groups should share their methods and solutions for the practical activity, and suggest ways in which the task might be refined. They could also make their own suggestions for different practical activities using trigonometry.

Summary for students

- Trigonometry is used to solve problems with right-angled triangles.
- Pythagoras' theorem is used to solve problems with right-angled triangles.
- Trigonometry can be used to calculate lengths and angles that might be difficult, or impossible, to measure directly.
- Trigonometry is used extensively by surveyors, astronomers, engineers and navigators.

10.3

Advanced

Objectives

Congruent triangles

- Use knowledge of angles at a point, angles on a straight line, and alternate and corresponding angles between parallel lines and a transversal line to present formal arguments to establish the congruency of two triangles.
- Establish the congruency of two triangles to generate further knowledge and theorems about triangles.
- Understand similarity of two triangles.

Starter

Vocabulary

congruent triangles

Resources

OHT 10.3a, 10.3b

Thin straight sticks of varying lengths, some cards cut to make wedges with specified angles

The lesson will be the first of several dealing with congruence and similarity. Its theme is an introduction to the idea of congruence.

It is assumed that students can draw straight lines accurate to 1 mm and angles accurate to 1° , as during the lesson they will need to draw some triangles with specified sides and angles.

Start the lesson by showing **OHT 10.3a** and explain that the slide shows how to annotate triangles so that they can be named and talked about with precision: An upper-case letter is used for the angle and lower-case of the same letter for the side opposite that angle (or full specification of the angle or side by moving through the angle or side from one vertex to another). Students must adopt these conventions because, in this and the lessons that follow, the importance of specifying every bit of the triangle unambiguously will be paramount.

Initiate a discussion with a question:

Q What is the least number of measurements to describe a triangle uniquely? What measurements would you need?

Allow students access to the resource materials to try to make triangles for themselves. Can they do it with one side alone? With one angle alone? With two sides or two angles? With one side and one angle?

Q Can you, for example, draw a triangle with one side 5 cm and one angle 37° so that all the class draws exactly the same triangle? Why not?

Now ask students to draw accurately a triangle with sides 5 cm, 8 cm and 10 cm. Show **OHT 10.3b**, explaining how it shows the necessary construction with arcs of circles to make this triangle. Pass the overlay around for all students to compare their own triangle with the overlay. Check that, within limits of accuracy, they all agree that their triangle is the same shape and size as the one on the transparency.

Tell students that they have demonstrated that they can construct a triangle given the lengths of three sides and that all these triangles are congruent to each other. Define the word *congruent* as having the same shape (angles match) and the same size (lengths of sides match). Add that their demonstration would seem to indicate that the triangle with sides 5 cm, 8 cm and 10 cm is unique; there is only one such triangle in terms of its properties. Now pose (but do not expect an answer to) the leading question:

Q Will three measurements always be sufficient to specify a triangle uniquely?

Main activity

Vocabulary

similar triangles
proof
theorem
common
axiom

Resources

OHTs 10.3c, 10.3d
Blank acetate sheets (or
tracing paper)
Acetate pens
Squared paper

For the next part of the lesson (about 20 minutes), first get the students to suggest the other possibilities with just three measurements (two sides and an angle, two angles and a side, three angles). Now split the class into three groups.

- 1 Each student in group 1 is make an accurate drawing of any triangle with two specified sides and the 'included' angle between them (SAS).
- 2 Each student in group 2 to make an accurate drawing of any triangle with two specified angles and one specified side (AAS).
- 3 Each student in group 3 to make an accurate drawing of any triangle with three specified angles (AAA).

Each group should specify the measurements they will use. Every student in each group should consider whether it is possible to draw more than one triangle with the given measurements. One student in each group should trace their drawing on to an acetate sheet and all other members of a group should compare their own drawings with the trace on the acetate

Now have a class discussion. Ask each group in turn to present their findings, explain how they constructed their triangles, and be prepared to answer questions from any member of the class. The discussion should reveal that in the SAS and in the AAS groups, each student in a group produced a triangle that was congruent to everyone else's triangle.

Group 3 should have found that they could draw a variety of triangles of different sizes, but the same overall shape (determined by the three specified angles).

Explain that these triangles are *similar*, but not congruent.

Now ask

Q Have all the three measurement possibilities been considered?

Make sure that the special situation **RHS** (right angle, hypotenuse and side) is considered and instruct students to draw a right-angled triangle that you specify. Instruct the students to draw your triangle on squared paper and then hold up their drawings. It is easy to see that all the triangles will be congruent to each other.

Now show **OHT 10.3c** which illustrates two sides and the 'non-included' angle (**ASS**). Explain why these measurements are not a condition for congruency as it is possible to construct two different triangles from these measurements.

Summarise the four conditions for congruence (**OHT 10.3d**) and explain that these may now be taken as granted to deduce further facts. Such deductions are often called *proofs*, and an important results is often called a *theorem*.

In the next part of the lesson (about 20 minutes), use congruence of two triangles to work through two proofs with the active participation of students. This will begin to demonstrate the power of using congruence to establish new results.

Draw a parallelogram ABCD on the board and then the diagonal AC. Ask students what they notice. Someone will probably mention that:

In parallelogram ABCD, $\triangle ABC$ is congruent to $\triangle ADC$.

If so, ask

Q Yes, they look congruent, but how can we be sure that they are congruent?

Draw out the steps of the proof with support from the students with questions like:

Q If the triangles are congruent, what pieces of information do we need to demonstrate to prove the congruence?

Build on what they already know about parallelograms and the angles a transversal makes with parallel lines to get them to suggest what they can use, or deduce, as facts to prove the congruence of the two triangles.

They might need help with using appropriate words, for example the side AC is *common* to both $\triangle ABC$ and $\triangle ADC$.

Once it has been established that $\triangle ABC$ is congruent to $\triangle ADC$, it follows that the diagonal of a parallelogram cuts the parallelogram into two equal halves, since congruent triangles are identical in every respect. It also follows that opposite sides of a parallelogram are equal to each other as well as being parallel to each other.

Quickly recapitulate the steps of the proof and emphasise that it is important to give a reason that justifies each step in the proof. This is to make certain that the result is correct.

Next, draw the remaining diagonal BD. In the same interactive way, prove together that:

The diagonals of a parallelogram bisect each other.

Further work

Complete exercises on matching triangles for congruence.

Construct further proofs using congruent triangles.

Understand the basis for geometric constructions using congruent triangles.

Consolidation

Q What are the four ways of establishing congruence?

Q Why does ASS not establish congruence?

Q When are triangles similar but not congruent?

Have a brief discussion about how agreed facts (or *axioms*) can be used to deduce other results.

Summary for students

- Two triangles are congruent if they are identical in shape and size.
- Three measurements are necessary to establish congruence of two triangles.
- The sets of three measurements that establish congruence of two triangles are SSS, SAS, RHS and AAS.
- Congruence is used to prove new facts from agreed facts.

10.4

Advanced

Objectives

Exploring data sets

- Collect meaningful primary data from representative samples in order to test hypotheses about, or estimate characteristics of the population as a whole.
- Formulate and solve problems using secondary data from published sources, including the Internet.
- In analysing data, calculate and use measures of central tendency such as the arithmetic mean and the median.
- Calculate measures of spread, including the variance and standard deviation.
- Construct histograms, grouping continuous data when necessary.
- Plot cumulative frequency distributions, grouping continuous data when necessary.
- Draw stem-and-leaf diagrams and box-and-whisker plots and use them in presentations of findings.

Starter

Vocabulary

data table
frequency table
outlier
histogram
cumulative frequency distribution
stem-and-leaf diagram
box-and-whisker plot

Resources

OHTs 10.4a, 10.4b

The purpose of this lesson is to get students to work with substantial data sets, and to display and analyse the data in various ways, using representations that they have already learned about.

Start the lesson by showing **OHT 10.4a**. This slide shows two ways of displaying data. Two more ways are shown on **OHT 10.4b**.

Q In what other ways can data be displayed?

Q How is data classified? Give an example with each response.

Q What is a stem-and-leaf diagram? What advantages are there in using such diagrams?

Q Why is a box-and-whisker plot sometimes called a five-number summary?

Q Why is a cumulative frequency distribution useful?

Q What is an outlier? Why are outliers important?

After a few refresher questions of this sort, including quick recall of some relevant and related vocabulary (from the list below), begin the main activity of the lesson.

Main activity

Vocabulary

percentile
quartile
median
interquartile range
outlier
mean
standard deviation
variance

Resources

Statistical software package and/or graphics calculators

Collecting the data

The purpose of this extended activity is to get students to work with substantial data sets, and to display and analyse the data in various ways. You will need to decide whether students should collect their own primary data or use secondary data, large amounts of which can be drawn from the Internet.

The activities in this lesson are probably best done in small groups, whose members should decide for themselves how to approach the work and assign the various subtasks that need to be carried out. The work will involve working with data about the heights of students and their feet sizes.

Describe the outline of the task you decide to set for the students and tell them whether you want them to collect primary data in the class or school, or to use secondary data from the CaS database, or some mixture of both.

The CaS contains much data about the height and foot size of children of different ages at schools in the different countries in the project, and from different economic backgrounds. Samples of children with their height and size measurements can be drawn randomly from the database. In this way random size samples of say 30 (or even 200, or however many) children of the same age, in the same or different countries may be compared with each other. Comparisons can also be made by gender and using other categorisations.

Using primary data collection in class, students will have to collect this data from other students in the school, finding some method of random selection to choose each sample. Collect as many samples as seems reasonable in your school, bearing in mind that groups of students in your class will have to analyse the sample data.

Students should not waste time by doing arduous calculations. Use ICT to do the calculations, and to generate the graphs and tabulations.

Analysing and representing the data

Use data on student height (in cm) and foot size (in cm) to illustrate how data may be displayed, compared and analysed.

This is an open-ended task. Advise students to decide on an aim or purpose for their investigation and that they need to make sure that this is included in the write-up. Also advise them to consider for inclusion in their work:

- how the data (either primary or secondary) was collected or selected, and reference to the source of any secondary data that is used;
- the size of the sample(s) with a reason for the choice of sample size;
- the data used and any assumptions made about it;
- the variety of different graphs/charts that can be used to represent continuous distributions, and the grouping of data if this seems necessary;
- the nature of the outliers in the data, and how to deal with the outliers;
- the different types of numbers that can be used to represent or characterise continuous distributions;
- sample statistics;
- how to compare two different distributions of the same type, and which to compare;
- the important words to use in describing their findings;
- whether there appears to be any correlation between the variables, and if so what sorts of conclusions might be suggested;
- speculation about what might affect the variables under consideration.

Help students with any difficulties that they have with analysing or representing data. Give feedback on how any aspect of the work could be improved or clarified.

Further work

The main activity can lead to many further activities. Some possibilities might be:

- comparisons between different countries in CaS;
- describing a modal pupil;
- calculating probabilities connected with the data used in the main task;
- modelling some of the data with a mathematical model, for example seeking a mathematical relationship between foot size and height;

- setting other tasks based on different variables;
- beginning to build up a database for your own school, using data gathered by your classes and do further analysis on pooled data.

Consolidation

Each group should give a short resume of what they have done.

Help them to discuss any difficulties they encountered and get the groups themselves to resolve difficulties by making constructive suggestions.

Ask what they found most useful aspect of their investigation, giving reasons why.

Q Which representations did you find most useful for comparisons?

Q Which was most useful for displaying the raw data? Why?

Q How did you deal with outliers?

Q How would you do things better the next time?

Q How could you extend the work?

Summary for students

- Decide what the project is about and what questions need to be answered.
- Decide what data will be useful, and how it will be collected.
- Use appropriate representations to present the data and make comparisons.
- Consider the significance of outliers.
- Calculate relevant statistics for the data.
- Use correct terminology in presenting findings.

Note on sources of data

Three websites from which secondary data are available readily are:

- www.censusatschool.ntu.ac.uk
- www.census.gov
- www.statistics.gov.uk

The first site is a very interesting one for teachers, and may be freely sampled. The data on this website are collected from over 800 000 responses from schools in the United Kingdom, South Africa, Queensland (Australia), New Zealand and Canada. The *CensusAtSchool* (CaS) project (run by the Royal Statistical Society Centre for Statistical Education in the United Kingdom) welcomes schools in other countries anywhere around the world to join (at no cost) and add to its growing international database by responding to the census questionnaire. Qatar could become yet another country participating in the project. The website contains details of useful projects and activities for students, the international database, a random selector from the database, details on how to use and extract data into a spreadsheet and much else that is extremely useful. Make sure that you familiarise yourself with this site if you intend to use information from it in your classroom. The lesson builds on the assumption that CaS can be used to enhance the tasks set to students, but it is not essential.

The second website is the United States Census. The third one is the Office of National Statistics in the UK.

You should also be familiar with local sources of statistical information in Qatar.