

## Spot the pattern

<b>C0</b>	<b>C1</b>	<b>C2</b>	<b>C3</b>	<b>C4</b>	<b>C5</b>	
0	5	10	15	20	25	<b>Row 5</b>
0	4	8	12	16	20	<b>Row 4</b>
0	3	6	9	12	15	<b>Row 3</b>
0	2	4	6	8	10	<b>Row 2</b>
0	1	2	3	4	5	<b>Row 1</b>
<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>Reference</b>

### Construction rule

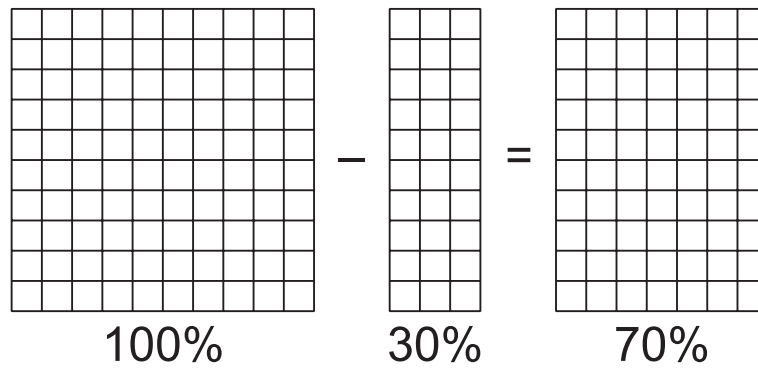
What are the patterns to go right or left along each row?

What are the patterns to move up or down each column (excluding the entries in the reference row)?

## Multiplication of integers

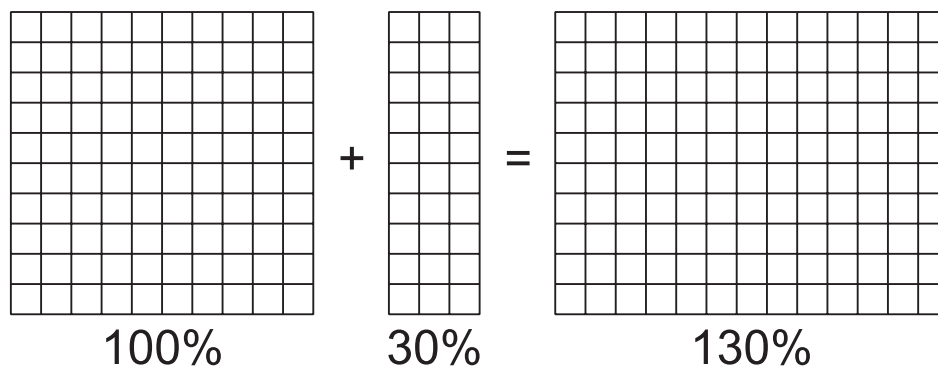
-25	-20	-15	-10	-5	<b>5</b>	0	5	10	15	20	25
-20	-16	-12	-8	-4	<b>4</b>	0	4	8	12	16	20
-15	-12	-9	-6	-3	<b>3</b>	0	3	6	9	12	15
-10	-8	-6	-4	-2	<b>2</b>	0	2	4	6	8	10
-5	-4	-3	-2	-1	<b>1</b>	0	1	2	3	4	5
0	0	0	0	0	<b>0</b>	0	0	0	0	0	0
<b>-5</b>	<b>-4</b>	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>×</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
5	4	3	2	1	<b>-1</b>	0	-1	-2	-3	-4	-5
10	8	6	4	2	<b>-2</b>	0	-2	-4	-6	-8	-10
15	12	9	6	3	<b>-3</b>	0	-3	-6	-9	-12	-15
20	16	12	8	4	<b>-4</b>	0	-4	-8	-12	-16	-20
25	20	15	10	5	<b>-5</b>	0	-5	-10	-15	-20	-25

## Percentage decrease



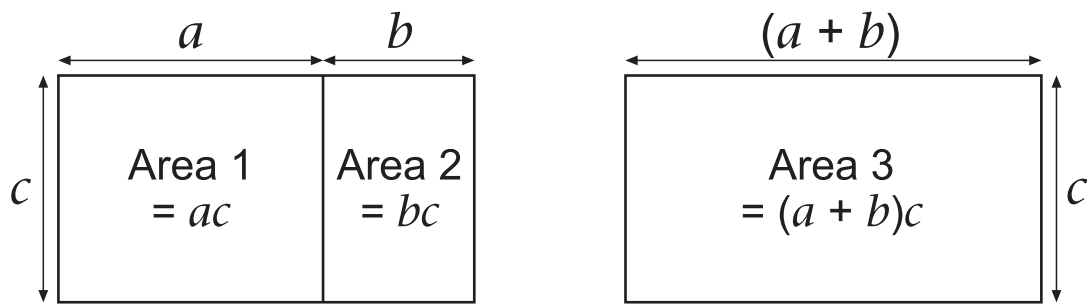
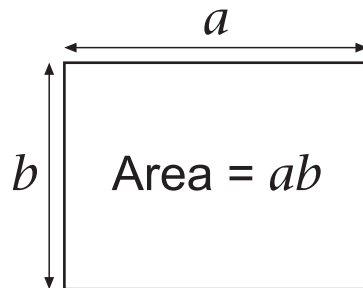
$$\begin{aligned} A - 30\% \text{ of } A &= 70\% \text{ of } A \\ &= 0.7A \end{aligned}$$

## Percentage increase

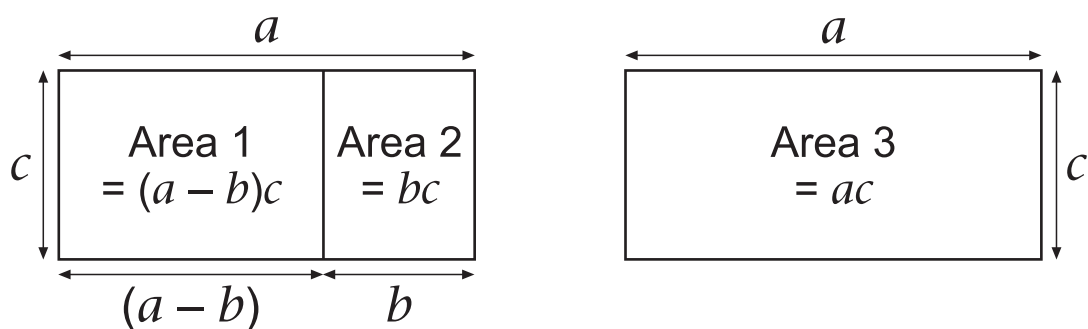


$$\begin{aligned} A + 30\% \text{ of } A &= 130\% \text{ of } A \\ &= 1.3A \end{aligned}$$

## Multiplication as area 1

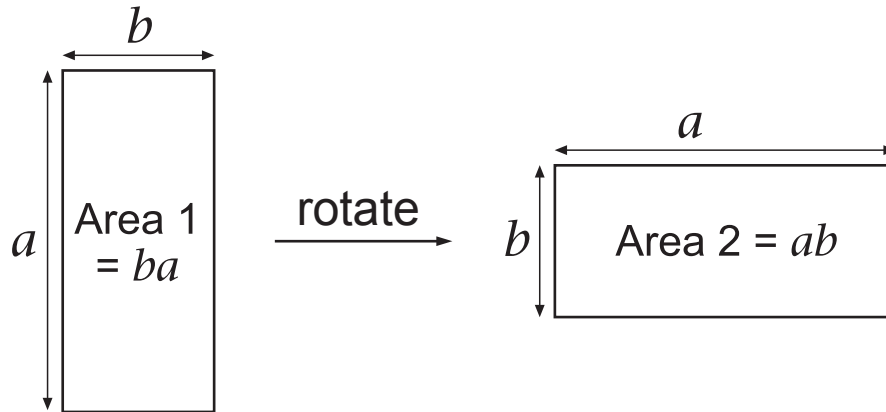


$$\begin{aligned} \text{Area 1} + \text{Area 2} &= \text{Area 3} \\ ac + bc &= (a + b)c \end{aligned}$$



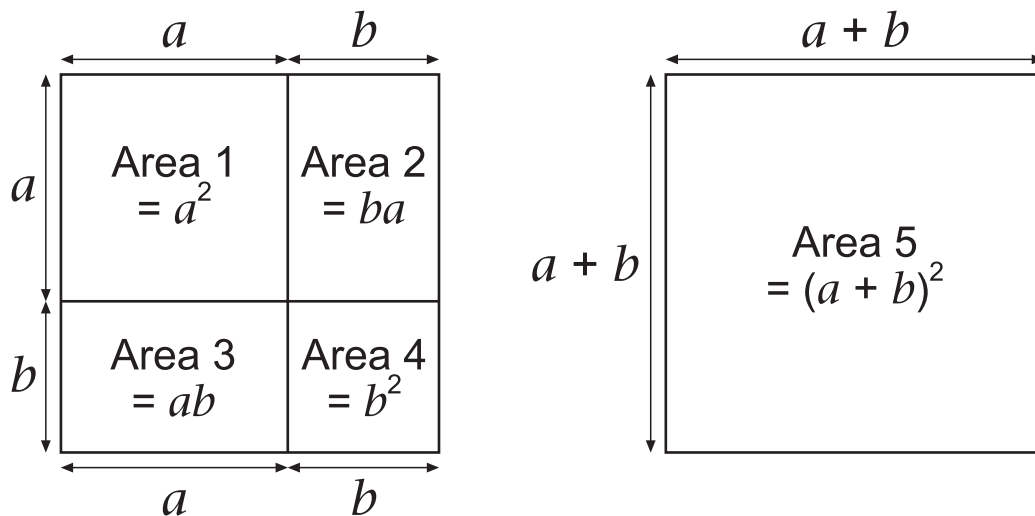
$$\begin{aligned} \text{Area 1} &= \text{Area 3} - \text{Area 2} \\ (a - b)c &= ac - bc \end{aligned}$$

## Multiplication as area 2



$$\text{Area 1} = \text{Area 2}$$

$$ba = ab$$



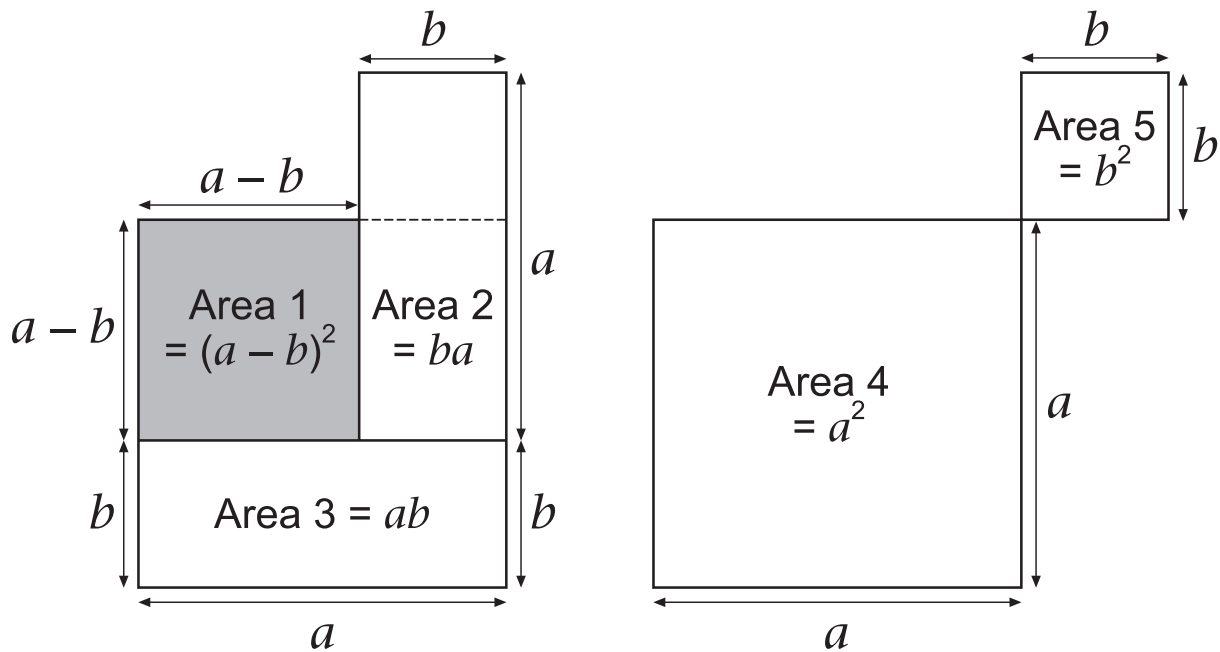
$$\text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4} = \text{Area 5}$$

$$a^2 + ba + ab + b^2 = (a + b)^2$$

$$a^2 + ab + ab + b^2 = (a + b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

### Multiplication as area 3

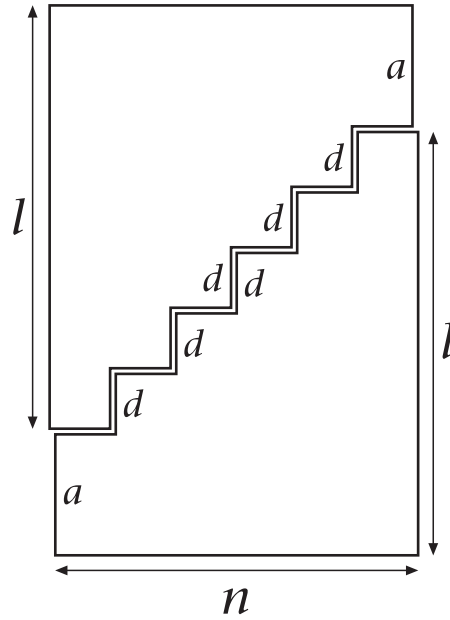


$$\text{Area 1} = \text{Area 4} + \text{Area 5} - \text{Area 2} - \text{Area 3}$$

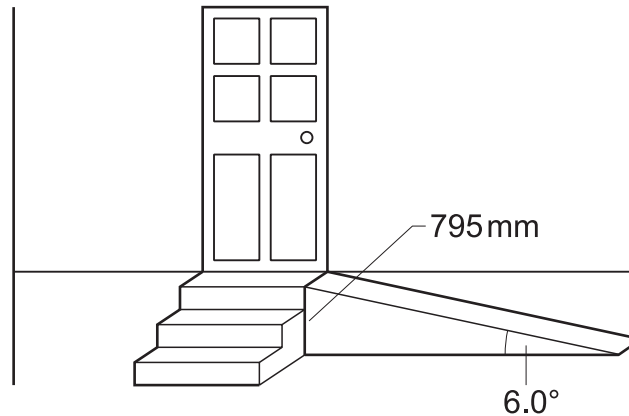
$$(a - b)^2 = a^2 + b^2 - ba - ab$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

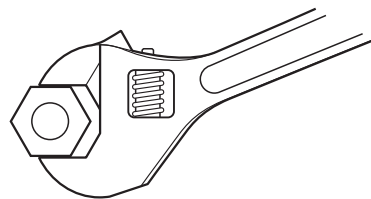
## Summing an arithmetic series



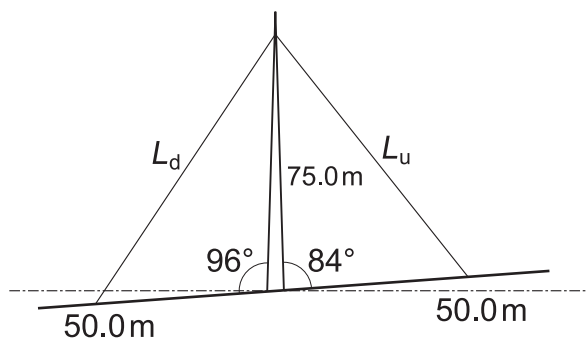
- 1 Mahmoud is designing a building in which a doorway is 795 mm above the ground. A ramp for the disabled, at an angle of  $6.0^\circ$  with the ground, is to be built to the doorway. How long should the ramp be?



- 2 A nut is in the shape of a regular hexagon. The length of each side is 9.53 mm. What opening on a wrench is necessary to tighten the nut?

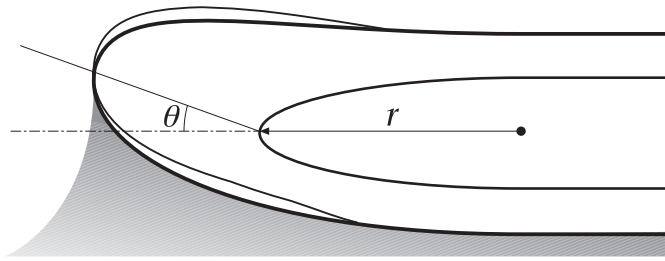


- 3 A vertical radio antenna is to be built on a hill that makes an angle of  $6.0^\circ$  with the horizontal. Guy wires are to be attached at a point 75.0 metres up the antenna and at points 50.0 metres from the base of the antenna. What will be the lengths of the guy wires positioned directly up and directly down the hill?



- 4 When the Sun's altitude is  $55^\circ$ , a flag-pole casts a shadow of length 25.3 metres. How high is the flag-pole?

- 5 The curves on a car race track are banked, as shown in the diagram, for safety reasons. Engineers also design banks on curves on public roads and railroads for the same reason.



The angle of the bank depends on the expected speed of vehicles using it. The road should be banked at an angle  $\theta$  according to the equation

$$\tan \theta = v^2 / gr$$

where  $v$  is the speed of the car and  $r$  is the radius of the curve in the road.

Use the formula to find  $\tan \theta$  and hence the angle  $\theta$  for the curve with radius of 282 m if the vehicle speed is  $v = 27.8$  m/s and  $g = 9.80$  m/s<sup>2</sup>.

The width of the road at the bank is 25 metres. How much higher is the left-hand side of the bank than its right-hand side?

- 6 A hot air balloon is rising vertically. An observer at ground level makes two observations one minute apart and records that the angle of elevation of the top of the balloon changes from  $35^\circ$  to  $45^\circ$ . The observer is a horizontal distance of 1000 metres from where the balloon began its ascent. How high does the balloon rise in the minute between observations? Give the answer to the nearest 10 metres.
- 7 Two ships leave harbour at the same time. The first ship sails on a bearing  $050^\circ$  at a speed of 12 kilometres per hour. The second ship sails on a bearing of  $30^\circ$  at a speed of 16 kilometres per hour. How far apart are the boats after 4 hours? Give the answer to the nearest kilometre.
- 8 A radio mast stands on top of a building 150 metres high. A surveyor at ground level a horizontal distance of 200 metres from the base of the building observes the mast and notes that the difference in angle of elevation between the top and bottom of the mast is  $9^\circ$ . Find the height of the mast.

## Answers to exercise

(Questions 7 and 8 are more difficult than the others.)

1 7.610 m

2 16.5 mm

3  $L_u = 87.5$  m,  $L_d = 94.4$  m

4 36.1 m

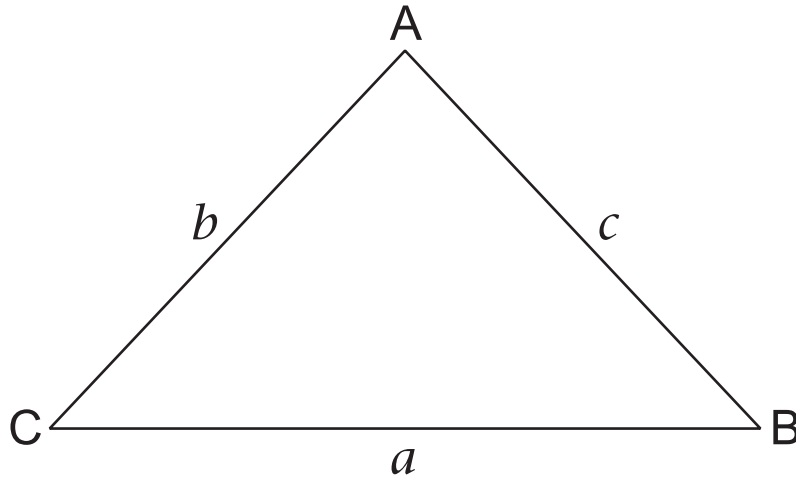
5  $\theta = 15.61^\circ$ ; left side is 6.73 m above right side.

6 300 metres

7 25 km

8 56.17 m

## Notation for triangles



The triangle is denoted by its vertices as  $\triangle ABC$

Side AB or  $c$  (since it is opposite vertex C)

Side AC or  $b$  (since it is opposite vertex B)

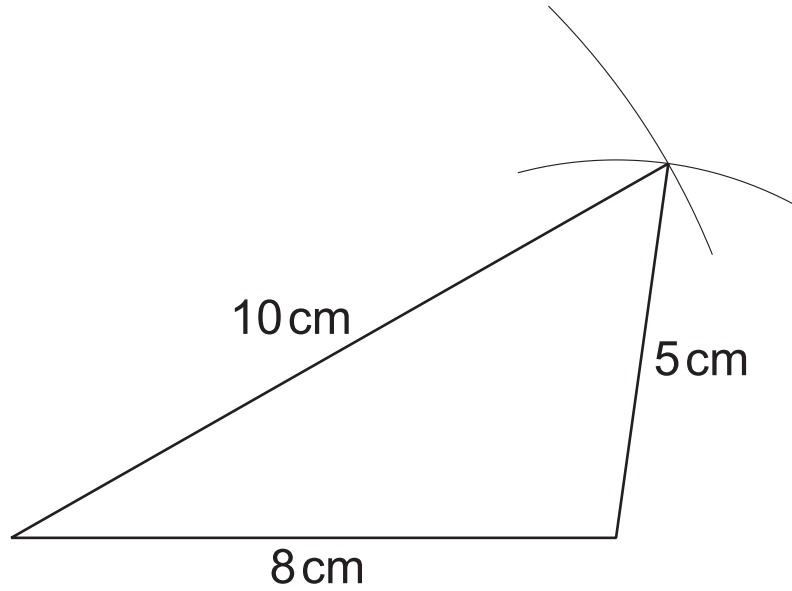
Side BC or  $a$  (since it is opposite vertex A)

Angle A (if this is unambiguous) or  $\angle BAC$

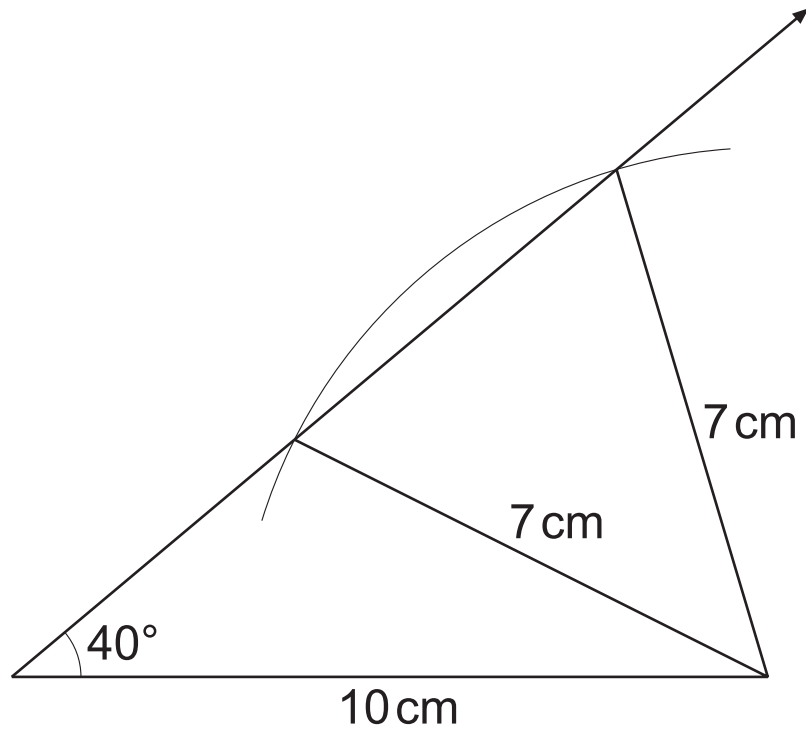
Angle B (if this is unambiguous) or  $\angle ABC$

Angle C (if this is unambiguous) or  $\angle ACB$

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**Triangle with three given sides**

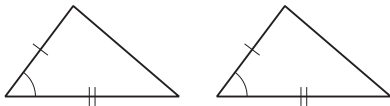
## The ambiguous case ASS



## The four conditions for congruence

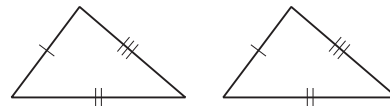
### SAS

If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent.



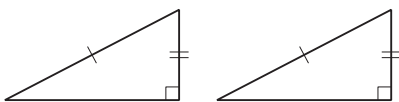
### SSS

If the three sides of one triangle are respectively equal to the three sides of another triangle, then the two triangles are congruent.



### RHS

If the hypotenuse and a side of one right-angled triangle are respectively equal to the hypotenuse and side of another right-angled triangle, then the two triangles are congruent.



### AAS

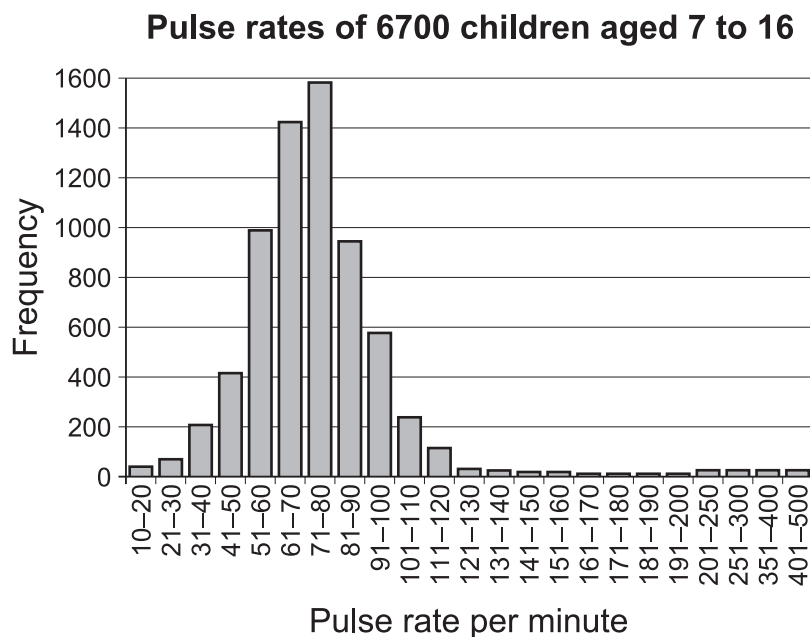
If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle, then the two triangles are congruent.



## Table of animal pulse rates

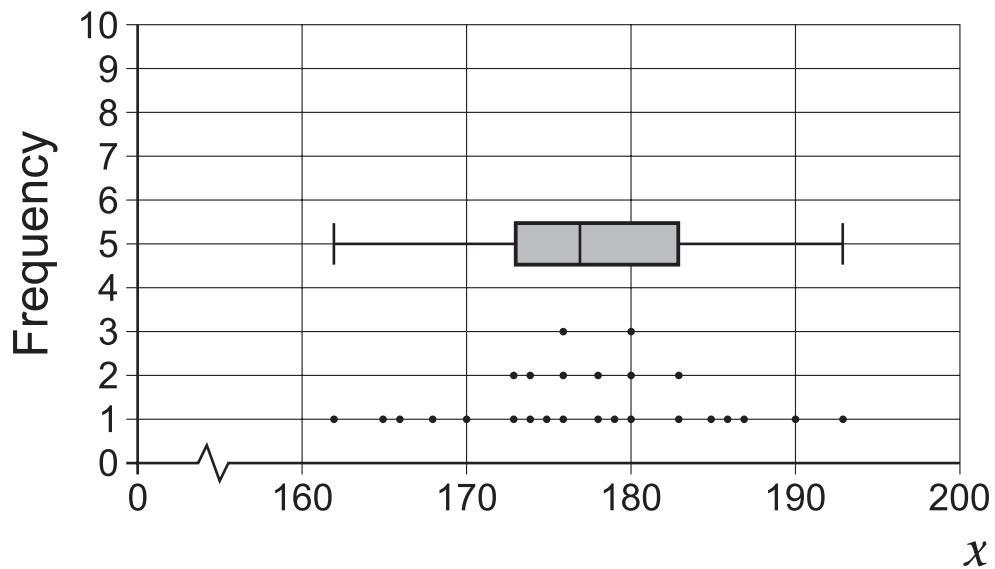
Type of animal	Normal pulse rate
Horse	32–44
Dog	60–120
Sheep	70–80
7–16 year olds	75–100
Mouse	450–550
Elephant	20–30
Newborn baby	130–140

## Histogram



Data from Phase 3 CensusAtSchool

## Box-and-whisker plot and dot plot



The dot plot shows the actual data values.

## Statistics for data set used in the diagram above

Number in sample, $n$	26
Mean, $\bar{x}$	177.308
Standard deviation, $\sigma_x$	7.46669
Range	31
Lower quartile	173
Median	177
Upper quartile	183
Semi-interquartile range	5