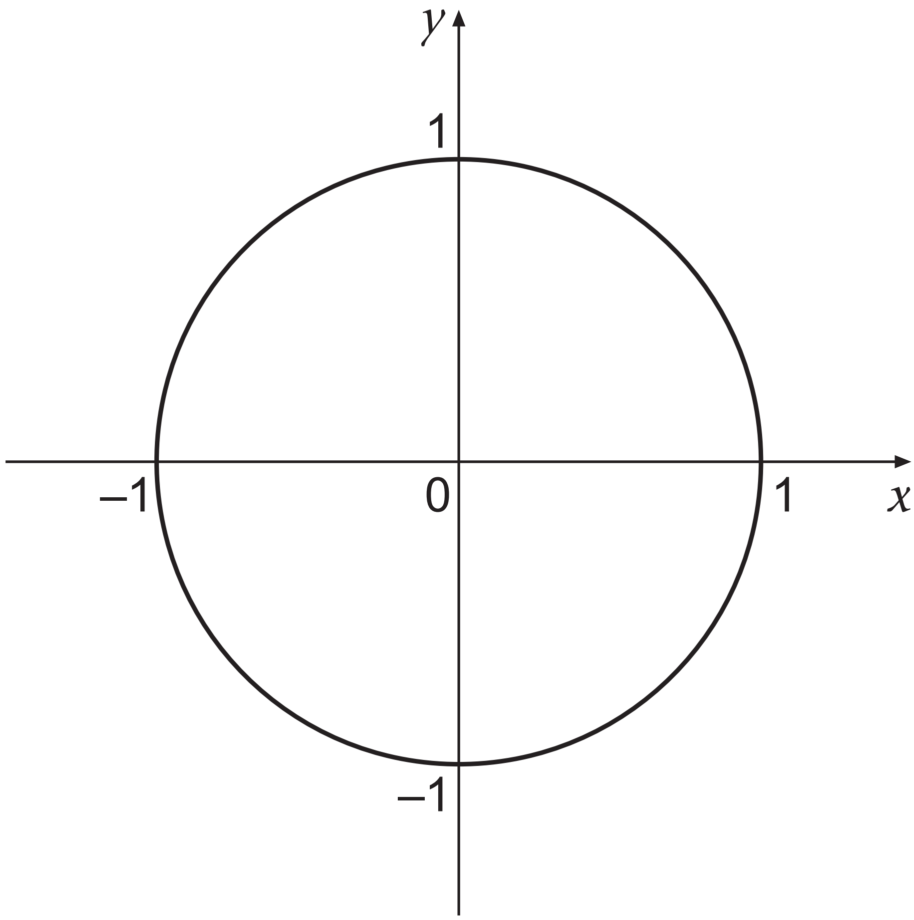
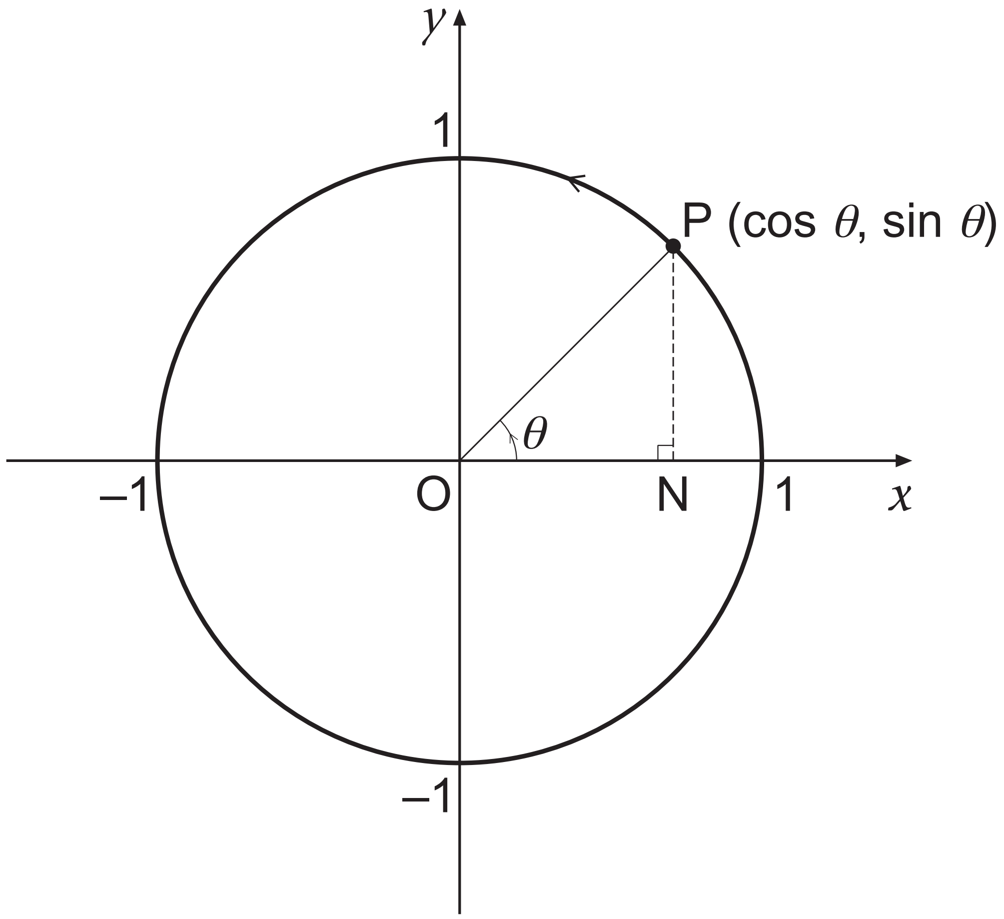
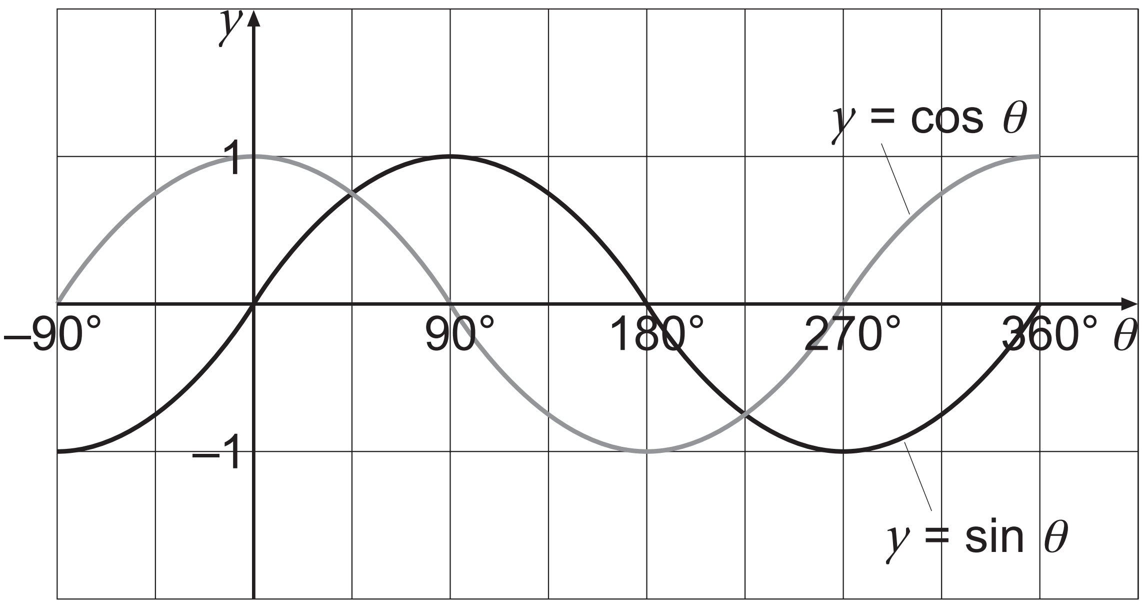
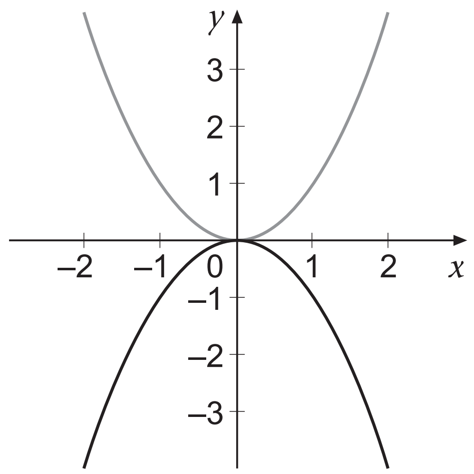
The unit circle

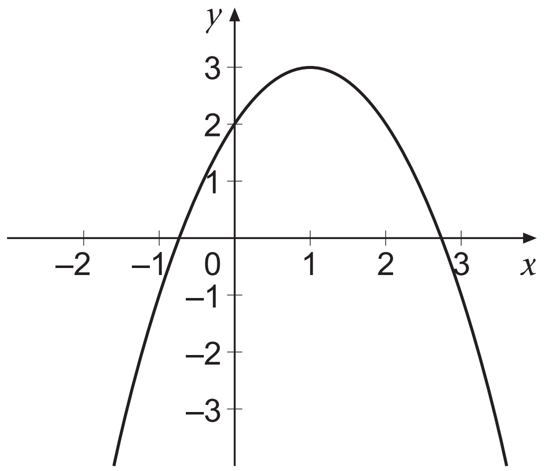


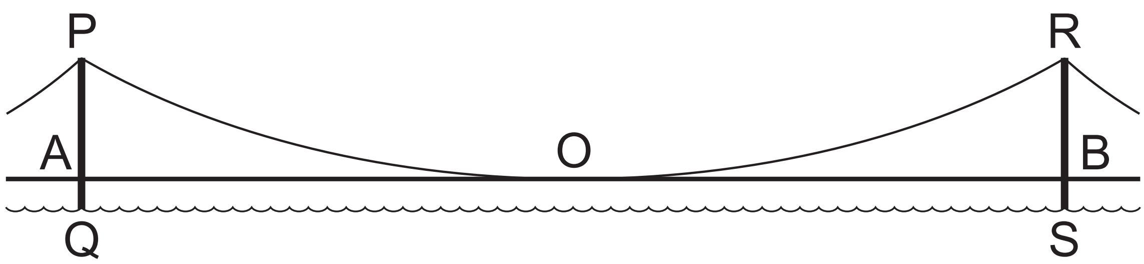

y
1
P (cos θ, sin θ)
θ
–1
O
N
1
x
–1

Graphs of the sine and cosine functions




y
3
2
1
–2
–1
0
1
2
x
–1
–2
–3


y
3
2
1
–2
–1
0
1
2
3
x
–1
–2
–3



The bridge goes over a sea bay.

Cable POR hangs in a curve that is a parabola.

AB is 16 times the length of PA.

The roadway is 60 metres above the sea.

Questions

1 Take the length PA as 1 unit. With this scale, show that the equation of the parabola formed by the hanging cable is $y = \frac{1}{64}x^2$.

2 The actual span AB of the bridge is 3200 metres. How high is the supporting frame PA?

3 When x and y are both measured in metres, what is the actual equation of the cable?

4 A workman is working on the cable halfway between its highest point and its lowest point. A very strong wind causes the workman to fall into the sea. How far does he fall?

Here are some population statistics concerning the ages and marital status of women in an un-named country (counted in 1000s of women).

Status	Age in years			Total
	16–30	**31–60**	**61 or more**	
Married	653	3651	689	4993
Widowed	4	209	699	912
Divorced	57	767	106	930
Never married	1161	599	63	1823
Total	1875	5226	1557	**8658**

A woman is chosen at random from this population.

Let A be the even that a woman of age 31–60 is chosen. Then $P(A) = \frac{5226}{8658} = 0.6036$.

Let B be the event that the woman chosen is widowed. Then $P(B) = \frac{912}{8658} = 0.1053$.

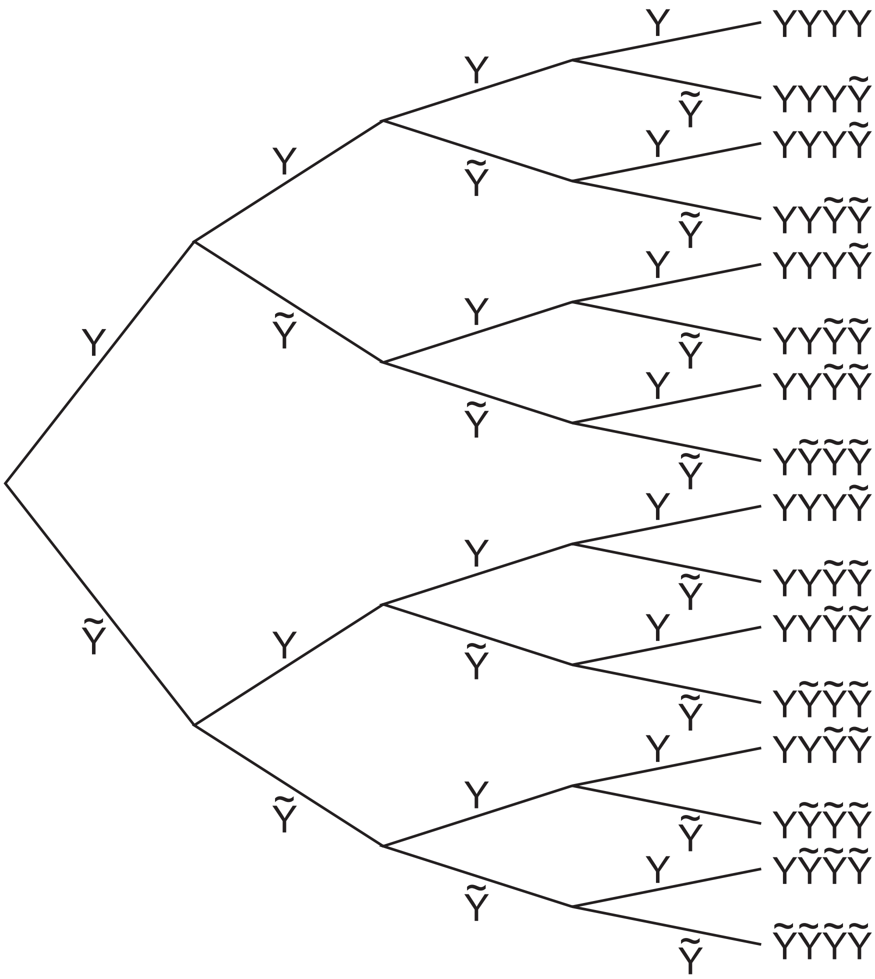
The probability that the chosen woman is both widowed and aged 31–60 is P(A and B). So $P(A \cap B) = \frac{209}{8658} = 0.0241$.

But the multiplication rule gives
$P(A \cap B) = P(A) \times P(B|A) = \frac{5226}{8658} \times \frac{209}{5226} = \frac{209}{8658}$ as above.

Resource 11.3b

Tree diagram

Drawing balls from a bag



Y is the event that a yellow ball is chosen

Ỹ is the event that a yellow ball is not chosen

Proof that all numbers in any finite set are interesting

Step 1 Assume the contrary: not all numbers are interesting.

Step 2 Then there is one non-interesting number that is the least non-interesting number. So, this number is interesting because it has this special property.

Step 3 Now remove this number from the set. Consider the remaining numbers in the set. Either the remaining numbers are all interesting or there is at least one non-interesting number in the remaining set.

Step 4 Repeat steps 2 and 3 until all the numbers in the original set have been dealt with.

Step 5 Once all the numbers in the set have been dealt with, it has been demonstrated that they are all interesting, although some of the numbers are non-interesting.

Conclusion There is a contradiction that can only be removed by changing the original premise and making all numbers interesting.

Examples of logic statements and notation

Implication

x is a multiple of 4 $\Rightarrow$ x is even
(x is even **if** x is a multiple of 4.)

Note that the converse is not true:
'x is even' does not imply 'x is a multiple of 4'.

If A $\Rightarrow$ B then A is a **sufficient** condition for B.
If A $\Leftarrow$ B then A is a **necessary** condition for B.

Equivalence

$x - 3 = 0 \Leftrightarrow x = 3$
($x - 3 = 0$ **if and only if** $x = 3$)

If A $\Leftrightarrow$ B then A is a **necessary and sufficient** condition for B.

more than 10 people are here $\Leftrightarrow$ at least 11 people are here

If A $\Leftrightarrow$ B then A is **equivalent** to B.

Only if

$x = \cos\frac{3\pi}{2}$ only if $0 \le x + 1 \le 1$

Consolidation exercise

1 Find counter-examples to disprove the following.

a. $x^3 \geq x$

b. The product of an odd and an even number is never a perfect square.

c. The sum of two irrational numbers is always irrational.

d. The product of two irrational numbers is always rational.

2 Which of the following are true statements?

a. $x^2 + x + 41$ is prime for all x.

b. The sum of two primes is prime.

c. $x^2 + y^2 \geq 2xy$

d. $p + q = 1 \Rightarrow p^2 + q = q^2 + p$

e. $x^2 + x \leq 9x - 17$

3 Correct the faulty logic.

$$\begin{aligned} s = 4 &\Rightarrow s^2 + 5s = 36 \\ &\Rightarrow s^2 - 4s = 36 - 9s \\ &\Rightarrow s(s-4) = 9(4-s) \\ &\Rightarrow s = -9 \\ \therefore -9 = 4 & \end{aligned}$$

4 Fill in the missing words or sign between these propositions.

a. x is even ……… x is a multiple of 6

b. The diagonals of a quadrilateral bisect each other
……… The quadrilateral is a rhombus

c. $\tan\theta = 1$ ……… $\theta = \dfrac{5\pi}{4}$

5 Prove (by induction) that $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$

for integer $n \geq 2$.

6 Use induction to prove a formula for the sum of the first n positive odd numbers.