Mathematics lessons for Grade 8

Lessons in this section

Using these lesson plans

These sample lessons for Grade 8 are suitable for use with a whole class. The lessons are single examples to illustrate different teaching and learning activities. They are not intended to be taught as a sequence. They are drawn from different topics and points in the teaching year to show spread rather than sequence.

The objectives for the lessons are drawn from the standards for Grade 8. Occasionally, a standard from an earlier grade is revised. The relevant standards are shown in the lesson plans.

The lessons are organised in three parts: a starter to introduce the lesson, a main activity, and a final phase to help students to reflect on the lesson and consolidate their learning. Before the starter, you should outline the purpose of the lesson, drawing out for students what they will learn and how this builds on previous work. In the final part of the lesson, you will need to establish the key learning points, what students need to remember and what they will go on to learn next. There is no expectation that students should copy out the key learning points in their exercise books.

The lesson plans do not include homework tasks because the lessons are single examples taken out of sequence. You will need to provide this, since homework is an important part of a lesson.

Each lesson plan has enough material to support about 45 minutes of teaching. You may need to supplement the activities with simpler or more challenging tasks if the students in your class have a range of attainment. You could choose from activities in textbooks or from your own resources. If you wish, different tasks can be given to different groups of students, according to their needs.

There may be too much material in the lesson plan for 45 minutes, since this will depend on the class. In this case, you could designate one of the activities in the lesson as homework, or carry it forward to the next lesson. Be selective about which activity to cut – it does not have to be the last one merely because it comes at the end.

Answers to questions are provided to help you to correct students' responses and give feedback. Sometimes, alternative answers are possible that are equally correct.

8.1 Powers and roots

Objectives

- Estimate and calculate positive integer powers of whole numbers and decimals; know cube roots of perfect cubes to ±216; use the cube root sign $\sqrt[3]{}$; find approximate values of square roots of whole numbers to 100.
- Use the x^2, $\sqrt{x}$ and x^y keys of a scientific calculator.

Starter

Vocabulary

square
square root

Resources

Mini-whiteboards
OHT 8.1a

Remind students that a square root of a positive number can be positive or negative, so that if $a = 9$, $\sqrt{a} = +3$ or –3, which can be written as ±3.

Show the grid of algebraic expressions on **OHT 8.1a**.

Give values to a and b. These need to be perfect squares: for example, $a = 9$, $b = 25$. Write these values on the board.

Point to an expression on the grid. Ask the class to evaluate the expression mentally and to write the answer on their mini-whiteboards. Choose a student with the correct answer to explain how to calculate the answer.

After pointing to several different expressions, change the values for a and b. For example, make $a = 1$ and $b = 4$.

Main activity

Vocabulary

method

Resources

Calculators
Computer with spreadsheet, and data projector

Tell the class that to cube a number is to raise it to the power of three. Four cubed is written as $4^3 = 4 \times 4 \times 4$.

Ask the class:

Q If $a = 1$, what is a^3? If $a = 2$, what is a^3? If $a = 3$, what is a^3? If $a = 4$, what is a^3? If $a = 5$, what is a^3?

As each answer is obtained, generate a sequence on the board:

1, 8, 27, 64, 125, …

Continue with:

Q If $a = 6$, what is a^3? If $a = 7$, what is a^3? If $a = 8$, what is a^3? If $a = 9$, what is a^3? If $a = 10$, what is a^3?

Encourage students to work out the answers mentally. Remind them of mental strategies such as:

$36 \times 6 = (30 \times 6) + (6 \times 6)$
$49 \times 7 = (50 \times 7) – (1 \times 7)$
$81 \times 9 = (81 \times 10) – (81 \times 1)$

Continue the sequence on the board:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

Q If $a^3 = 729$, what is a?

It is possible that answers of +9 and –9 will be given. Stress that only one answer is possible for the equation $a^3 = 729$. Demonstrate that

$–9 \times –9 \times –9 = –729$

Introduce the cube root notation. Tell the class that if 729 is the cube of 9, then 9 is the cube root of 729, which is written as $\sqrt[3]{729} = 9$. Write on the board:

What is $\sqrt[3]{64}$? What is $\sqrt[3]{125}$?

Ask the students what 2^4 means, and what its value is. Repeat with 3^5, continuing to encourage mental calculation.

Extend the questioning to decimals. For example, ask students to calculate:

$(0.1)^3$, $(2.5)^2$, $\sqrt[3]{0.027}$, $\sqrt{0.81}$

Show students how to use their scientific calculators to find squares, square roots, cubes and cube roots. Explain that, depending on the type of calculator that they are using, sometimes the square root sign is typed before the number and sometimes afterwards. Ask students to use their calculators to find:

$\sqrt{12.25}$ (answer 3.5)
$\sqrt{33\,124}$ (answer: 182)

Extend the questioning to negative numbers. Ask students to calculate the following, using their calculators where they need to:

$(–8)^2$, $(–2)^5$, $(–4)^4$, $(–5)^3$

Record answers on the board. Ask the class what they notice. Draw out that even powers of negative numbers are positive, and odd powers of negative numbers are negative.

Q What is $(–1)^{223}$?

Q What is $(–1)^{224}$?

Ask the class how they could find $\sqrt{7}$ if they had only a basic calculator with no square root key. Work with the class to find $\sqrt{7}$ to two decimal places.

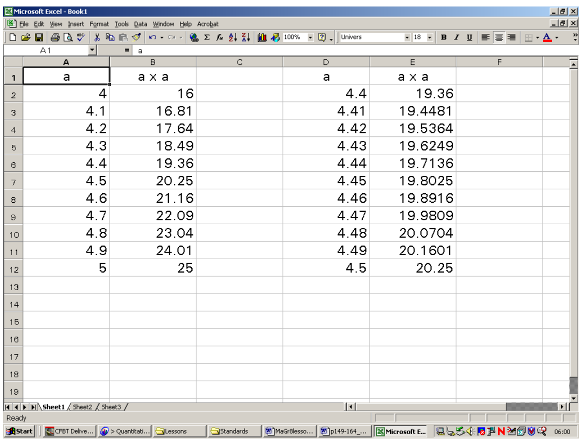
Explain that $\sqrt{7}$ must lie between 2 and 3, because $\sqrt{4} = 2$ and $\sqrt{9} = 3$.

Try $2.5^2 = 6.25$. This is too low.
Try $2.6^2 = 6.76$. This is too low.
Try $2.7^2 = 7.29$. This is too high.
Try $2.65^2 = 7.0225$. This is very close but a little bit too high.
Try $2.64^2 = 6.9696$. This is very close but too low.

The answer is $\sqrt{7} = 2.65$ to two decimal places.

Ask students to work in pairs and using only the × key on their calculator to find $\sqrt{20}$ to two decimal places (answer: 4.47).

Show the class how they could use a computer spreadsheet for this activity



a	a x a		a	a x a
4	16		4.4	19.36
4.1	16.81		4.41	19.4481
4.2	17.64		4.42	19.5364
4.3	18.49		4.43	19.6249
4.4	19.36		4.44	19.7136
4.5	20.25		4.45	19.8025
4.6	21.16		4.46	19.8916
4.7	22.09		4.47	19.9809
4.8	23.04		4.48	20.0704
4.9	24.01		4.49	20.1601
5	25		4.5	20.25

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Bring the whole class together.

Ask the class:

Q How can we write $a \times a \times a$ concisely? (a^3)

Stress the different meanings of the two expressions $3a$ and a^3.

Write some examples on the board for students to simplify by writing the answers on their whiteboards:

$m \times m \times m \times m \times m$
$2a \times a$
$4y \times 3y \times 2y$
$s + s + s + s + s \times s \times s \times s$
$b(4b - 2)$

Discuss the factorisation of an expression such as $24x^3 + 18x^2$.

One possible factorisation is $2(12x^3 + 9x^2)$. Ask students to work in pairs and to find all the other possible factorisations.

Round off the lesson by summarising the key points for students.

Summary for students

- a^2 means $a \times a$, and a^3 means $a \times a \times a$.
- $\sqrt{a}$ means the square root of a, and $\sqrt[3]{a}$ means the cube root of a.
- Positive numbers have two square roots, e.g. $\sqrt{6.25} = +2.5$ or -2.5.
- Even powers of negative numbers are positive, and odd powers of negative numbers are negative.

8.2 Sequences

Objectives

- Extend and find missing terms in numeric, algebraic or geometric patterns or sequences (term-to-term or position-to-term rules).
- Generalise the relationship between one term of a sequence and the next, or describe the nth term, using symbols.
- Choose and use appropriate mathematical techniques to solve a problem.
- Use diagrams and explanatory text to explain the solution to a problem and support it with evidence.

Starter

Vocabulary

sequence
term

Resources

Mini-whiteboards

Tell the class that the lesson is about sequences. Write on the board:

The first term is ☐.
To find the next term of the sequence, add ☐.

Invite a student to choose a number to go in each box (e.g. 4 in the first box, 3 in the second box).

Q What are the first five terms of this sequence? (4, 7, 10, 13, 16)

Now ask, a series of questions, drawing out the possibilities where there is more then one answer.

Q What numbers would you put in the boxes to make:

- **all the numbers in the sequence even?**
- **all the numbers in the sequence odd?**
- **all the numbers in the sequence multiples of 3?**
- **all the numbers in the sequence end in the same digit?**
- **exactly ten two-digit numbers in the sequence?**
- **every other number in the sequence a whole number?**
- **every fourth number in the sequence a whole number?**
- **every fourth number in the sequence a multiple of 5?**

Main activity

Vocabulary

formula
quadratic sequence
first difference
second difference

Resources

Copies of Resource 8.2a, one per student

Write on the board $T(n) = 2n + 7$.

Explain that the notation $T(n)$ stands for the nth term of a sequence. It is a formula describing a sequence, from which every term can be found by substituting the integers 1, 2, 3, 4, …

Working with the class, generate the sequence: 9, 11, 13, 15, … Point out that the sequence goes up in 2s and that the first term is 2 + 7.

Repeat with $T(n) = 3n + 2$.

This generates 5, 8, 11, 14, … Point out that this sequence goes up in multiples of 3, the number in front of the n, plus 2. Show the class that this rule will also work for negative numbers.

Repeat with $T(n) = 4n – 3$. This sequence is 1, 5, 9, 13, …

Now write on the board: 9, 13, 17, 21, …

Ask the class:

Q What is the *n*th term is for this sequence?

Ask the students for the reasons why they have suggested various rules. Lead them to the fact that, since 4 is added on every time to obtain the next term, the nth term will start with $4n$.

Then consider the first term, which is of the form $4n + d$, where $n = 1$. See what has to be added to 4 to get the first term of 9. This will be 5. So the nth term will be $4n + 5$.

Write on the board the sequence 3, 8, 13, 18, … Ask:

Q What is the *n*th term is for this sequence?

Students should be able to identify the first part as $5n$ since 5 is added on each time. Ask:

Q What must be added to 5 to get the first term of 3? (–2)

Establish that the nth term is given by $5n – 2$.

Write on the board the sequence 1, 4, 9, 16, 25, 36, … Ask:

Q What do you notice about the differences between consecutive terms?

After some discussion, put on the board the first and second differences:

	1		4		9		16		25		36
First differences		3		5		7		9		11	
Second differences			2		2		2		2		

Explain that when a sequence has the same second differences, it is a *quadratic sequence* – that is, a sequence whose nth term contains n^2. In the example above, $T(n) = n^2$.

Finally, discuss the sequence $T(n) = n(n + 2)$, and show again that the second differences are constant.

Give out copies of **Resource 8.2a**, one per student. Ask students to complete the questions in their exercise books, working independently.

Circulate as students work, assisting any who need help, and checking answers.

Answers

1 $2n + 1$, the 17th term

2 $n^2 + 4$, the 9th term

3 a. $4n + 2$
b. $7n + 1$
c. $3 – 4n$
d. $3n – 18$
e. $\frac{(2n+1)}{(5n-1)}$

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Resources

Mini-whiteboards

Bring the whole class together. Write on the board, leaving plenty of space between each term:

5, 11, 17, 23, …

Q What are the next three terms of this sequence? (29, 35, 41)

Q Is it possible to insert a number between each pair of numbers of this sequence so that there is still a simple rule? (5, 8, 11, 14, 17, 20, 23, … making the rule change from 'add 6' to 'add 3')

Ask students to work in pairs and to discuss their answers with a partner before giving them to the whole class.

Repeat the last question for each of these sequences, writing each of them on the board.

a. 5, 20, 80, 30, …

b. 9, 81, 729, 6561, …

c. 3, 6, 12, 24, 48, …

Answers:

a. Change the rule from 'multiply by 4' to 'multiply by 2'.

b. Change the rule from 'multiply by 9' to 'multiply by 3'.

c. Change the rule from 'multiply by 2' to 'multiply by $\sqrt{2}$'.

Summary for students

- Sequences are sometimes defined by a formula for the nth term, e.g. $T(n) = 2n + 1$. The terms of the sequence can be generated by substituting into the formula the values 1, 2, 3, 4, … for n.
- Sequences are sometimes defined by a rule which relates each term to the previous term, e.g. $T(n) = T(n - 1) + 5$, an 'add 5' rule. If you know the first term, you can work out the second term; if you know the second term, you can work out the third term, and so on.

8.3 Interior and exterior angles of polygons

Objectives

- Use knowledge of the angle, side and symmetry properties of triangles to conjecture or deduce properties in a given figure.
- Calculate interior and exterior angles of polygons.
- Use step-by-step reasoning to deduce properties or relationships in a given geometrical figure.

Starter

Vocabulary

triangle
congruent

Resources

Copies of Resource 8.3a, one per student
Pencils
Computer with dynamic geometry system (DGS), such as Geometer's Sketchpad, and data projector

Ask students to work in pairs. Give out **Resource 8.3a**, one for each student.

Tell the class that in this activity they are given a number of descriptions of triangles. Their task is to sketch triangles with these descriptions if it is possible to do so. They should label the triangles with the given lengths of sides and angles. Stress these points about the descriptions.

- Some describe just one possible triangle.
- Some descriptions are impossible.
- Some describe more than one possible triangle. In this case, they should sketch the variations.

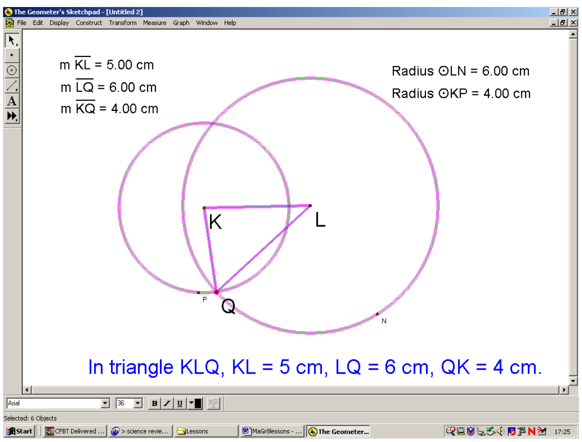
Tell the pairs that they should discuss each case before they sketch it.

Allow about 10 minutes for students to talk about and sketch the triangles. Circulate as they do so, making sure that they consider alternative possibilities in relevant cases. Then ask:

Q **Which triangles are impossible?** (4 and 8) **Explain why.** (in 4, the sum of the lengths of the two shortest sides is less than the length of longer side; in 8, the sum of the three angles is greater than 180°)

Q **Which descriptions fit more than one possible triangle?** (2 and 3)

Discuss each description and draw it using a dynamic geometry system, or sketch it on the board.



Stress that in the cases where only one triangle is possible, all the people who drew a triangle with this description would draw identical or *congruent* triangles.

Main activity

Vocabulary

polygon
interior angle
exterior angle
vertex/vertices

Resources

Computer with DGS and data projector

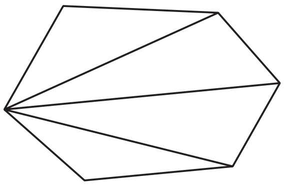
Copies of Resource 8.3b, one per student

Remind the class that the sum of the *interior angles* of a triangle is 180°, and that the sum of the interior angles of a quadrilateral is 360°. Illustrate by splitting a quadrilateral into two triangles.

Remind students also that a *polygon* is a plane shape with three or more straight sides. A *regular polygon* has sides of equal length.

Q What other names of polygons do you know? (pentagon, hexagon, heptagon, octagon, nonagon, decagon, …)

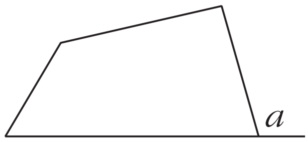
Demonstrate on the board or using a dynamic geometry system how to find the sum of the *interior angles* of a hexagon. Show how a hexagon can be split into four triangles from one of its vertices.



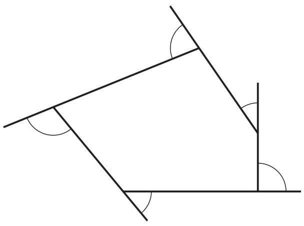
Explain that the sum of the interior angles of each triangle is 180°. So the sum of the interior angles of a hexagon is 180° × 4 = 720°.

From this we can deduce that each interior angle of a regular hexagon is 720° ÷ 6 = 120°.

Explain to the class how to form an *exterior angle* of a polygon by extending a side of the polygon. In the diagram, *a* is an exterior angle of the quadrilateral.



Extend all the sides of a pentagon to give all its exterior angles.



Explain how, if a person stands on a vertex and turns through all the exterior angles in turn, she or he will have turned through 360°. So the sum of the exterior angles is 360°. Stress that this is true for all polygons.

From this we can deduce that each exterior angle of a regular pentagon is 360° ÷ 5 = 72°.

Ask students:

Q What is the size of the interior angles of a regular nonagon? (140°)

Q What is the size of the exterior angle of a regular nonagon? (40°) **Why?** (the exterior angle and the interior angle have a sum of 180°)

Give out copies of **Resource 8.3b**, one per student. Ask students to complete the questions.

Answers to the questions are:

1 112.5°

2 100°

3 18

4 20

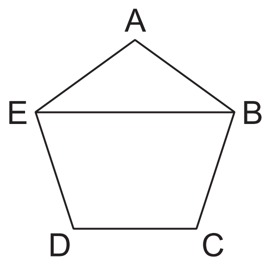
5 The interior angle and exterior angle at each vertex sum to 180°.
The total of the three exterior angles and interior angles is 180° × 3 = 540°.
The three interior angles have a sum of 180°.
So the three exterior angles have a sum of 540° – 180° = 360°.

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Draw on the board a regular pentagon ABCDE.



Ask students to work in pairs and to calculate ∠ABE. Invite a pair to the board to explain each step of their reasoning.

Repeat with a regular hexagon.

Able students may be able to show that, given a regular polygon with *n* sides, when the first and third of three adjacent vertices are joined, the base angle of the isosceles triangle that is formed is $\frac{180°}{n}$.

Summary for students

- Congruent triangles are identical in size and shape.
- The angles inside a polygon are known as the interior angles.
- If we extend the side of a polygon, the angle formed outside the polygon is known as an exterior angle.
- At any vertex of any polygon, the interior angle and the exterior angle form a straight line, so that their sum is 180°.
- The sum of the exterior angles of any polygon, regular or irregular, is 360°.

8.4 Probability

Objectives

- Use data from experiments to estimate probability for favourable outcomes.
- Understand that different outcomes may result from repeating an experiment.
- Use problem conditions to calculate theoretical probabilities for possible outcomes.
- Compare experimental and theoretical probability in simple cases.

Starter

Vocabulary

equally likely
chance

Resources

A dice
Mini-whiteboards
Two tables already prepared on the board or an OHT (see the lesson notes)

Hold up the dice. Tell the class that you are going to throw it 24 times and see how many times you get a one, a two, a three, a four, a five and a six. Ask students to use their whiteboards to answer your questions.

Q Make a guess. Which number do you think I will throw most often?

Q Make another guess. How many times in 24 throws do you think that I will throw your number?

Draw this table on the board or show it on an OHT.

Dice number	Tally	Total
1		
2		
3		
4		
5		
6		

Choose a student to record results, then throw the dice 24 times as quickly as is reasonable. Write in the totals, and discuss the results.

Q Which number had the most throws? Is this what you expected?

Q Imagine throwing the dice another 24 times. Would we get the same results?

Repeat the experiment, using the second copy of the table to record results, and confirm that they are different.

Establish that when the dice is thrown, any of the six numbers is equally likely to appear. They all have the same chance. Because there are six numbers on the dice, each number should appear in about one sixth of the throws.

Q What is one sixth of 24? (4)

Stress that after 24 throws, we would estimate each number to have appeared 4 times. The theoretical probability is 4 out of 24. Compare this with the results of the experiment. Point out that the experimental and theoretical probabilities do not necessarily match.

Main activity

Vocabulary

fair, unfair
biased

Resources

Mini-whiteboards

A dice and 18 counters for each pair of students

OHT 8.4a

Draw this table on the board.

Dice number	Tally	Total
odd		
even		

Say that this time you will see how many even numbers you throw.

Q Make a guess. Which do you think I will throw more often: odd numbers or even numbers?

Q Make another guess. How many times in 30 throws do you think that I will throw an even number?

Discuss some of the guesses. Establish that, as half of the numbers on the dice are even, one of them should appear in about one half of the throws. The theoretical probability of throwing an even number is 1 in 2, or ½.

Ask students to copy the table from the board. Give each pair of students a dice. Ask one of the pair to roll the dice 15 times, and for the other to tally the result as 'odd' or 'even'. Then change over for another 15 throws. Finally, they should count the tally marks to find the total for each of 'odd' and 'even'.

Q What happened? How many times in 30 throws did you throw an even number?

Stress again that the experimental and theoretical probabilities may differ.

Ask students what they think a *fair game* is. Establish that it is a game which each player has an equal chance of winning. Explain that a *fair dice* is one in which each number has an equal chance of being rolled, and that a *fair coin* is one that has an equal chance of landing heads up or tails up.

Ask the pairs to play this game three times and to record who wins.

- Each player starts with 9 counters.
- Players take turns to roll the dice.
- The first player wins odds and the second player wins evens.
- If 1 or 3 or 5 is rolled, evens has to give to odds that number of counters.
- If 2 or 4 or 6 is rolled, odds has to give to evens that number of counters.
- The winner is the first to gain all the counters.

Q How many games did odds win? How many games did evens win? Is this a fair game? Why not?

Establish that each number on the dice is equally likely to be rolled. In six throws, each number is likely to be rolled once. Odds would in three times, and would win 9 counters. Evens would win three times, and would win 12 counters. The game is *unfair* because it is *biased* in favour of evens.

Remind the class that a probability scale is usually numbered with 0 at one end (impossible) and 1 at the other end (certain). A probability is usually written as a fraction (and sometimes as a decimal or percentage).

Show **OHT 8.4a**. With the class, complete the table. For example, in the first row, the possible numbers on the dice are 2, 4 and 6. There are three possible numbers, and the probability of throwing one of them is 3⁄6 or ½. Invite a student to the projector to locate the probability on the probability scale.

Repeat with the other rows.

Q Someone throws a dice 24 times. How many times would you expect them to get a 5? (1 in every 6, or 4 times)

Q How many times would you expect them to get an even number? (1 in every 2, or 12 times)

Q How many times would you expect them to get a number bigger than 4? (2 in every 6, or 8 times)

Other tasks

If necessary, choose further related activities, selecting from **Resource 8.4b**, available textbooks or your own materials.

Consolidation

Resources

OHTs 8.4c, 8.4d

Show **OHT 8.4c**. Discuss the bags of balls and the probability of choosing a black one.

Q What is this probability as a fraction? As a decimal?

Invite a student to mark the probability on the scale.

Show **OHT 8.4d**. Refer to the first problem and discuss the six faces of the dice.

Q How many faces does the dice have altogether? How many show a 2? How many show a 5?

Q What is the probability of rolling a 5? (2⁄6 or ⅓) **Of rolling a 2?** (4⁄6 or ⅔)

Invite another pupil to mark this probability on the scale.

Refer to the second problem and discuss the sectors of the spinner. Point out that they are not all equal. Some have a better chance than others.

Q What is the chance of the pointer landing in sector C? (about 1 in 4, or ¼)

Invite another pupil to mark this probability on the scale.

Q What is the chance of the pointer landing in sector E? Is it more than one half, or less than one half? (less than one half)

Invite a student to mark this probability on the scale.

Repeat for sector A.

Summary for students

- A probability scale has 0 at one end (impossible) and 1 at the other (certain).
- Probabilities are usually written as fractions or decimals, and sometimes as percentages. An even chance, or a one in two chance, is written as a probability of ½, 0.5 or 50%.
- A fair game is one in which each player has an equal chance of winning.

