Mathematics lessons for Grade 9

Lessons in this section

Using these lesson plans

These sample lessons for Grade 9 are suitable for use with a whole class. The lessons are single examples to illustrate different teaching and learning activities. They are not intended to be taught as a sequence. They are drawn from different topics and points in the teaching year to show spread rather than sequence.

The objectives for the lessons are drawn from the standards for Grade 9. Occasionally, a standard from an earlier grade is revised. The relevant standards are shown in the lesson plans.

The lessons are organised in three parts: a starter to introduce the lesson, a main activity, and a final phase to help students to reflect on the lesson and consolidate their learning. Before the starter, you should outline the purpose of the lesson, drawing out for students what they will learn and how this builds on previous work. In the final part of the lesson, you will need to establish the key learning points, what students need to remember and what they will go on to learn next. There is no expectation that students should copy out the key learning points in their exercise books.

The lesson plans do not include homework tasks because the lessons are single examples taken out of sequence. You will need to provide this, since homework is an important part of a lesson.

Each lesson plan has enough material to support about 45 minutes of teaching. You may need to supplement the activities with simpler or more challenging tasks if the students in your class have a range of attainment. You could choose from activities in textbooks or from your own resources. If you wish, different tasks can be given to different groups of students, according to their needs.

There may be too much material in the lesson plan for 45 minutes, since this will depend on the class. In this case, you could designate one of the activities in the lesson as homework, or carry it forward to the next lesson. Be selective about which activity to cut – it does not have to be the last one merely because it comes at the end.

Answers to questions are provided to help you to correct students' responses and give feedback. Sometimes, alternative answers are possible that are equally correct.

9.1 Standard form

Objectives

- Use index notation and the laws of indices to evaluate expressions with integral powers, including positive and negative powers of 10.
- Read and write numbers in the standard form $A \times 10^n$, where n is a positive or negative integer and $1 \le A < 10$; interpret numbers in standard form on a calculator display; use standard form in calculations and to estimate.
- Recognise when an exact solution to a problem is required and when an approximate solution is sufficient, and give answers to a specified degree of accuracy.

Starter

Vocabulary

base
power
index/indices, index form
exponent
million
billion
trillion
googol
nano
pico

Resources

OHT 9.1a
Mini-whiteboards

Write 10^2 on the board.

Q What does this represent? ('ten squared' or 'ten multiplied by ten' or 'one hundred')

Repeat with 10^3 ('ten cubed' or 'ten multiplied by ten multiplied by ten' or 'one hundred multiplied by ten' or 'one thousand').

Repeat with 10^4 ('ten to the power 4' or 'ten thousand'). Show students that, since $10^4 = 10 \times 10 \times 10 \times 10$, it can also be written as $10^3 \times 10$ or $10^2 \times 10^2$.

Now jump to 10^6.

Q What does this represent? ('ten to the power 6' or 'one million')

Ask students to suggest different ways of writing 10^6, e.g. $10^3 \times 10^3$ or $10^4 \times 10^2$.

Repeat with 10^9 ('ten to the power 9' or 'one thousand million' or 'one billion'), then 10^{12} ('ten to the power 12' or 'one million million' or 'one trillion').

Write 10^{100} on the board.

Q Does anyone know what this represents? ('a googol')

Explain that the words *index* (plural *indices*) and *exponent* mean the same as *power*. In the number 2^3, 2 is called the *base* and 3 is called the *power*. When a number is written in the form of base raised to a power, we say that it is written in *index form*. Explain that a negative index denotes a reciprocal, so that 2^{-3} indicates 1 divided by 2^3 , or one eighth, and 10^{-6} indicates 1 divided by 10^6, or one millionth.

Say that writing numbers in index form helps us to represent powers of numbers efficiently. Clarify that 10^0 has the value 1, and that 10^1 has the value 10. Students may also be interested in the names for 10^{-9}, one billionth (a nano), and 10^{-12}, one trillionth (a pico).

Remind the class of the rules for multiplying and dividing powers, giving one or two examples of each rule using small numbers.

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$
$(ab)^m = a^m b^m$

Show **OHT 9.3a**. Point to two different powers of 10 and ask students to multiply or divide them, writing answers on their whiteboards. Occasionally, point to just one power of 10 and ask a question such as:

Q What is this number raised to the power 5? To the power –3?

Main activity

Vocabulary

standard form

Resources

Scientific calculators

Students may have met the idea of *standard form* in science.

Explain that a way is needed to express very large and very small numbers without writing out lots of zeros. For example, 3 400 000 000 can be written as 3.4×10^9, and 0.000 072 can be written as 7.2×10^{-5}.

Ask students if they can see the connection with the original number and the power of 10.

Explain the definition of a standard form number:

$A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Demonstrate how to convert ordinary numbers to *standard form*, explaining that it is a matter of moving the digits and counting how many places that they move. For example:

$540\,000 = 5.4 \times 10^5$
$0.000\,005\,89 = 5.89 \times 10^{-6}$
$89\,630\,000 = 8.963 \times 10^7$

Now try examples such as:

$23 \times 10^6 = 2.3 \times 10 \times 10^6 = 2.3 \times 10^7$
$0.56 \times 10^{-4} = 5.6 \times 10^{-1} \times 10^{-4} = 5.6 \times 10^{-5}$

Repeat with more examples as necessary.

Now reverse the process. Ask the class to convert numbers in standard form to ordinary numbers. For example:

$3.45 \times 10^5 = 345\,000$
$8.6 \times 10^{-7} = 0.000\,000\,86$

Extend to multiplying two numbers each in standard form.

Write on the board: $(3 \times 10^6) \times (3 \times 10^2)$. Ask:

Q What is the answer?

Some students may respond intuitively that it is 9×10^8. Ask a student to explain the process of separating the numbers and the powers, or explain yourself:

$3 \times 3 \times 10^6 \times 10^2$

Now give the class: $(6 \times 10^6) \times (3 \times 10^2)$. Ask them for the answer. Some will probably respond 18×10^8. Ask:

Q What is wrong with this?

Explain that it is not in standard form and demonstrate how to amend it:

$18 \times 10^8 = (1.8 \times 10) \times 10^8 = 1.8 \times 10^9$

Repeat with: $(2.5 \times 10^{-2}) \times (8 \times 10^{-5})$. This gives:

$20 \times 10^{-7} = 2 \times 10 \times 10^{-7} = 2 \times 10^{-6}$.

Now give the class: $(2 \times 10^{-2}) \div (8 \times 10^{-5})$. Ask them for the answer. Demonstrate how to get it in standard form:

$$0.25 \times 10^3 = 2.5 \times 10^{-1} \times 10^3 = 2.5 \times 10^2$$

Next give the class: $(4.56 \times 10^3) \times (2.13 \times 10^{-7})$. They will realise that this is difficult without the use of a calculator. Explain how to use a calculator to enter numbers in standard form.

4.56×10^3 is entered as: [4] [·] [5] [6] [EXP] [3]

2.13×10^{-7} is entered as: [2] [·] [1] [3] [EXP] [7] [+/–]

Note that on some makes of calculator the appropriate key may be marked EE or with some other notation, and that the sign change key may operate differently.

Do the calculation above. The display should say 0.000 971 28 or 9.7128^{-04}. Convert this to the standard form number 9.7128×10^{-4}. If students are familiar with significant figures, round this to 9.71×10^{-4} (correct to three significant figures).

Repeat with: $(4.56 \times 10^3) \div (2.13 \times 10^{-7})$.

Make sure that students can enter this into their calculators using the EXP (or equivalent) key and the sign change key. The display should say $2.140\,845\,07^{\ 10}$. Convert this to a standard form number. If students are familiar with significant figures, round it to 2.14×10^{10} (correct to three significant figures).

Draw attention to the two errors that students tend to make with the use of calculators and numbers in standard form. Some students may enter 4.56×10^3 as:

[4] [·] [5] [6] [×] [1] [0] [EXP] [3]

This will give a value of 4.56×10^4.

The other error is to confuse a calculator display showing standard form as $4.3^{\ 2}$ with 4.3 raised to the power 2 rather than interpreting it as the actual standard form number 4.3×10^2.

Other tasks

Give students a related practice exercise, selecting from available textbooks or your own materials.

Consolidation

Give the class the problem: 0.012×0.006. They should be able to do this mentally and give the answer 0.000 072.

Ask them to 'translate' the calculation to a standard form problem:

$$(1.2 \times 10^{-2}) \times (6 \times 10^{-3}) = 7.2 \times 10^{-5}$$

Repeat with: $0.085 \times 0.04 = 0.0034$, or:

$$(8.5 \times 10^{-2}) \times (4 \times 10^{-2}) = 34 \times 10^{-4} = 3.4 \times 10^{-3}$$

Discuss the advantages of each method.

Summary for students

- A number in standard form takes the form $A \times 10^n$, where $1 \le A < 10$ and n is a positive or negative integer.

9.2 Graphs of functions

Objectives

- Find the gradients of lines given by $y = mx + c$; understand the idea of slope; find the gradients of lines parallel and perpendicular to $y = mx + c$.
- Represent, interpret, analyse and synthesise information presented in numeric, algebraic, geometric or graphical form.

Starter

Vocabulary

coordinates
rule
graph
equation
parallel
slope
gradient
translation

Resources

Resource 9.2a, one per student

Write on the board a list of coordinates:

(–5, –2) (1, 4) (5, 8) (30, 33) (45, 48)

Ask the class:

Q What is the rule that connects the *y*-coordinate to the *x*-coordinate? ($y = x + 3$)

Q What other coordinate pairs would fit the pattern?

Give out copies of **Resource 9.2a**, one per student. Ask students to plot the first three coordinate pairs and to join them, extending the line to draw the graph of $y = x + 3$.

Write on the board another list of coordinates:

(–3, 1) (0, 4) (2, 6) (5,9) (14, 18)

Q Can you see the rule this time? ($y = x + 4$)

As before, using the same coordinate grid, plot the first three coordinate pairs and draw the graph of $y = x + 4$.

Q What is the same and what is different about our two graphs?

Draw out that the two lines are parallel and have the same slope or gradient, but the second line has been translated by one unit in the positive direction of the *y*-axis.

Q What do you think the graph of $y = x + 6$ would look like?

Take suggestions.

Main activity

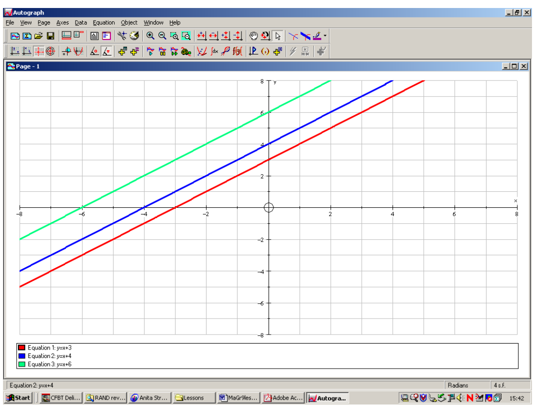
Vocabulary

linear
intercept
coefficient

Resources

Computer, graph plotting software and data projector
Graphics calculators
Resource 9.2b, one per student

Say that it would be quicker to draw the graphs using a computer and graph plotting software. Quickly enter the equations and draw the graphs of $y = x + 3$, $y = x + 4$, $y = x + 6$.



Point out the intercepts on the *y*-axis. Ask:

Q For graphs of equations of the form $y = x + c$, where c is a number, what is the connection between the intercept on the y-axis and the equation? (the constant c indicates the intercept on the y-axis)

Q What graph would pass through the point (0, –2) forming a line that is parallel to the three lines we have drawn? ($y = x - 2$)

Check the answer by adding the graph of this equation to the coordinate grid.

Clear the screen. Ask students to work in pairs and to use a graphics calculator to investigate graphs of equations of the form $y = mx$, where m is a number. Suggest that they try values of 1, 2 and 4 for m.

After a few moments, ask:

Q What is the same and what is different about your graphs? (they all pass through (0, 0) but the slopes vary)

Q What is the connection between positive values of the number m and the graph of $y = mx$? (m is the gradient or slope of the graph – the greater the value of m, the greater the slope)

Now ask students to try values for m of –1, –2, –4 on the same axes.

Q What is the connection between negative values of the number m and the graph of $y = mx$? (m is still the gradient or slope of the graph, but this time it slopes downwards from left to right)

Bring the whole class back together. Clear the screen again. Write on the board:

$y = 2x + 3$ $\qquad$ $y = x + 3$

Q What do you think would be the same and what would be different about the graphs of the equations $y = 2x + 3$ and $y = x + 3$?

Take suggestions. Test them out by drawing the two graphs on the screen. Establish that the graphs will both pass through the point (0, 3) on the y-axis but that the slope of $y = 2x + 3$ is greater than that of $y = x + 3$.

Tell the class that equations of the form $y = mx + c$ are called *linear* equations because the graphs of the equations are always straight lines.

Give out copies of **Resource 9.2b**, one per student. Ask students to work in small groups and to use their graphics calculators to help them to solve one or more of the problems.

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Resources

OHT 9.3c

Bring the whole class together. Write on the board $y = mx + c$. Ask the class:

Q What type of equation is this? (linear)

Q What is special about the graph drawn from an equation like this? (it's always a straight line)

Display **OHT 9.3c** (or set up similar graphs using graph plotting software, hiding the equations). Ask:

Q What are the equations of the lines in these two diagrams?

Ask students to explain their reasoning.

Answers

$$y = x + 1, \quad y = -x + 1$$

$$y = \frac{x}{2} + 2, \ y = \frac{x}{2} + 6, \ y = \frac{x}{2} - 2$$

Summary for students

- In a linear equation of the form $y = mx + c$, m and c stand for numbers. The values of m and c tell us where the graph of $y = mx + c$ is to be drawn.
- m is the gradient or slope and c is the intercept on the y-axis.

9.3 Similar triangles

Objectives

- Identify similar triangles and their corresponding angles and sides.
- Use the properties of congruence or similarity of triangles to solve problems, e.g. find unknown sides or angles of similar or congruent triangles.
- Develop a simple proof.

Starter

Vocabulary

equilateral
trapezium
isosceles
proof

Resources

Mini-whiteboards

Ask the class to close their eyes and to follow your instructions. Say:

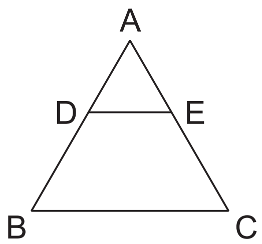
Imagine a large blue equilateral triangle on the table in front of you.

Place a small yellow equilateral triangle inside the large blue triangle.

Slide the small yellow triangle into a corner of the large blue triangle so that it fits exactly.

What shape is the blue shape that you can now see?

Encourage students to discuss their answers with a partner before writing them on their mini-whiteboards. Check answers. Sketch the diagram below on the board.



Now ask:

Q How do you know that the quadrilateral DECB is a trapezium?

Draw out the proof, writing it on the board.

$\angle ADE = \angle DBC = 60°$ (internal angles of equilateral triangles).

Since DE and BC are crossed by the transversal ADB, angles ADE and DBC are corresponding angles, and DE is parallel to BC.

Q How do you know that DECB is an isosceles trapezium?

AD = AE (sides of the same equilateral triangle ADE).

AB = AC (sides of the same equilateral triangle ABC).

So AB – AD = AC – AE, or DB = EC.

Main activity

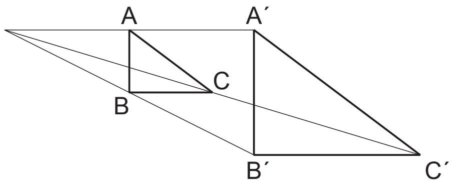
Vocabulary

map
enlargement
scale factor
similar

Resources

Resource 9.3a

Remind the class about the properties of an enlargement by showing them how a triangle is enlarged by a scale factor of 2.



Triangle ABC has been mapped onto triangle A′B′C′ by an enlargement of scale factor 2.

Under an enlargement all the angles are the same size and corresponding sides are in the same ratio.

So AB : A′B′ = AC : A′C′ = BC : B′C′ = 1 : 2.

This can also be written as $\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} = 2$.

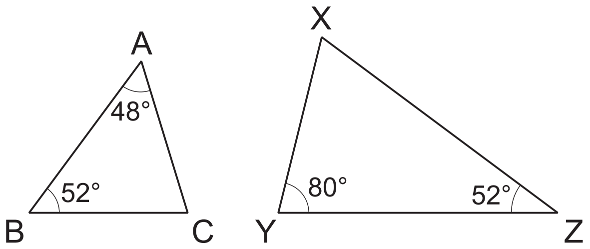
Explain to the class that the two triangles are said to be similar. Two triangles are similar if their angles are the same size or their corresponding sides are in the same ratio.

Stress that only one of these conditions is required to show that two triangles are similar.

Show the class how to use similar triangles by completing the following three examples. Some revision on parallel lines may be needed.

Example 1

Show that the two triangles below are similar.

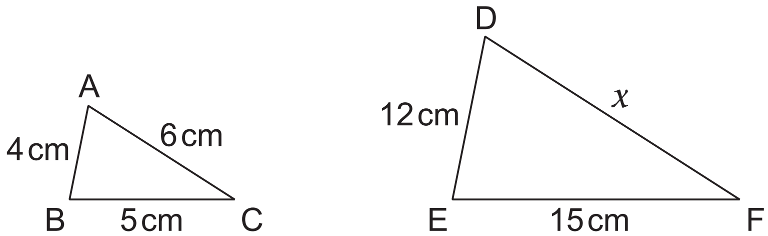


In triangle ABC, ∠C = 80° (the sum of the angles in a triangle = 180°) and in triangle XYZ, ∠X = 48° (the sum of the angles in a triangle = 180°).

Since the angles in both triangles are the same, triangle ABC is similar to triangle XYZ.

Example 2

Triangle ABC is similar to triangle DEF. Calculate the length of the side DF.



Let the side DF = x.

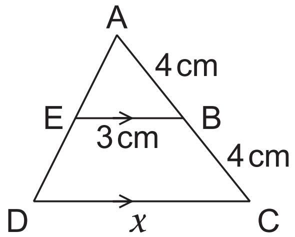
Since the triangles are similar, corresponding sides are in the same ratio.

So $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$.

Therefore, $\frac{x}{6} = \frac{15}{5} = 3$. So x = 18 cm.

Example 3

In the triangle, EB is parallel to DC. Calculate the length of the side DC.



$\angle$AEB = $\angle$ADC (corresponding angles in parallel lines).
$\angle$ABE = $\angle$ACD (corresponding angles in parallel lines).

So triangle AEB is similar to triangle ADC (since $\angle$A is common to both triangles).

Let the side DC = x.

Since triangle AEB is similar to triangle ADC, the corresponding sides are in the same ratio.

So $\frac{DC}{EB} = \frac{AC}{AB} = 2$.

Therefore, $\frac{x}{3} = \frac{8}{4} = 2$. So $x = 6$ cm.

Give out copies of **Resource 9.3a**, one per student, and ask students to complete the questions.

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Vocabulary

congruent

Resources

OHTs 9.3b and 9.3c

Bring the whole class together. Take feedback on the questions that students have done independently.

Work through one or both of the problems on **OHTs 9.3b** and **9.3c**.

Ask the class to explain the two conditions needed to show that two triangles are similar. Check that they understand the difference between similar triangles and congruent triangles.

Summary for students

- Under an enlargement all the angles are the same size and corresponding sides are in the same ratio.
- Two triangles are similar if their angles are the same size or their corresponding sides are in the same ratio.
- Two triangles are congruent if they are of the same shape and size, so that corresponding angles are equal and corresponding sides are equal.

9.4 Probability investigation

Objectives

- Use relative frequency as an estimate of probability and use this to compare outcomes of experiments.
- Compare experimental and theoretical probability in different contexts.

Starter

Vocabulary

trial
experimental probability
theoretical probability
relative frequency

Resources

None

Remind students that the *relative frequency* of an event is the number of times that it happens in a number of trials, and that relative frequency gives and estimate of probability.

For example, an experiment could be done to discover whether a dice is a fair dice or is biased. The dice is rolled 300 times and the number 1 comes up 36 times. The relative frequency of rolling 1 is $\frac{36}{300} = 0.12$. This is the *experimental probability*.

However, if the dice were unbiased, in theory the number 1 would come up once in every 6 rolls. In 300 rolls it could be expected to come up $\frac{1}{6} \times 300$ times, or 50 times. This is the *theoretical probability* of rolling 1 on a fair dice.

Discuss with the class whether they would conclude that the dice is fair or biased. Draw out that the results after 300 rolls may seem to indicate that the dice is biased but a greater number of trials would be needed before a definite conclusion could be drawn.

Main activity

Vocabulary

investigation
sample
fair
biased

Resources

OHTs 9.4a, 9.4b
Dice
Pack of playing cards
Open box(es)
Selection of cubes in at least four colours
Resource 9.4c

The activities given in this section could easily take two lessons, depending on the amount of detail that you ask for when students are carrying out the investigation. You may wish to ask the students to collect certain data before the lesson or you may decide to provide the students with secondary data.

Write on the board: 'Students in this school are better at probability than adults.'

Ask the class how they could investigate this statement. Draw out through discussion that one way would be to write a set of probability questions to be given to both students and adults. They could then record the results for their samples and compare the experimental probabilities of answering particular questions correctly for students and adults.

Discuss how they would decide which people to use in the sample.

Show **OHT 9.4a**, the data handling cycle, and **OHT 9.4b**, a checklist for completing a probability investigation. Go through these and discuss how each point would apply to the suggested investigation.

- **Statement of topic**
  Compare the abilities of students and adults at probability.
- **Hypothesis**
  'Students are better at working out theoretical probabilities than adults'.
- **Sample size**
  Look at 30 students and 30 adults.
- **Foreseen problems**
  Adults may be reluctant to answer the questions given.
  Choosing the sample may be difficult.

- **Sources of bias and how to minimise them**
  Avoid using all one age group for adults.
  It would not be sensible to use students in your class who have just revised probability.
- **Data collection**
  Record the number of correct and incorrect answers, the number of people who declined to do the questions and any other factors that may affect your results.
- **Extra information that may be required**
  Ask yourself the question: 'How can I extend this problem, using more complex techniques which will provide more reliable results?'
- **Analysis**
  Produce statistical diagrams to compare the success rates of students and adults.
  Calculate the experimental probability of each group answering a probability question correctly.
- **Limitations of any assumptions made**
  Students may have had a more recent experience than adults of work on probability, whereas adults may quickly understand the topic, if they were given a short briefing.
- **Conclusion**
  State whether, given your results, you agree with the initial hypothesis and why.

Ask students to work in groups to carry out the investigation on **Resource 8.4c**. In this experiment, a member of the group puts 10 coloured cubes in an open box so that the rest of the group do not know and cannot see what the colours are. The group investigates how many times they need to pick a cube out of the box and replace it to be able to predict accurately the colours of the cubes in the box.

Two further possible investigations are described below.

- Throw two dice. Add the two scores. How many different possible outcomes are there? Compare the experimental and theoretical probability of getting a total score that is:
  - even;
  - a multiple of 3;
  - a multiple of 4;
  - a multiple of 5;
  - a multiple of 6;
  - a multiple of 7.
- Throw a red dice and a blue dice. How many different possible outcomes are there? Compare the experimental and theoretical probability of these outcomes:
  - getting a 3 with the red dice and a 4 with the blue dice;
  - getting 5 on one of the dice and 6 on the other;
  - getting the same number on both dice;
  - getting a total score of 10 from the two dice.

Other tasks

If necessary, choose further related activities, selecting from available textbooks or your own materials.

Consolidation

Give one or more groups of students the opportunity to present their findings so far to the rest of the class. Encourage students to be critical of their own work. Help them to recognise the limitations of what they have done.

Discuss the effect of a small number of trials on the reliability of any relative frequencies as estimates of theoretical probability. Discuss how reliability might be improved, perhaps by increasing the size of the sample or by choosing the sample in a different way.

Discuss how the investigations could be extended.

Conclude the lesson by stressing how important probability is in real life. For example, weather forecasters try to warn people living in tropical regions of the approach of a hurricane, which tend to occur between July and October. The forecasters use probabilities to give an idea of the likelihood of a hurricane striking a particular area.

Summary for students

- Probabilities are numbers between 0 and 1.
- Relative frequency gives an estimate of probability.
- The probability of an event can be predicted by calculating how often it might occur in theory.
- The probability of an event can be estimated by carrying out an experiment and counting how many times the event occurs.
- An estimate of probability will be more reliable if the experiment is set up carefully to minimise possible bias.